Control of Unstable Bioreactor with Dominant Unstable Zero

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Original scientific paper
Received: September 12, 2002
Accepted: January 15, 2003

Design of a controller by synthesis method for an unstable bioreactor with a dominant unstable zero gives a PI controller. For such dominant unstable zero, it is shown that the integral time is negative. The stability analysis of such controlled system, is given. When the measurement delay is considered, the method requires a PI controller with negative integral time and a first order unstable filter. Theoretical analysis on the condition at which the system can be stabilized, is also given. Simulation results are given for the robust performance of the controller for perturbation in the controller settings. A set point weighting is proposed for the controller to reduce the initial jump in the response and also the undershoot.

Keywords:
PI controller, unstable system, dominant unstable zero, CSTR

Introduction

The performance of a controller is limited by the presence of unstable pole and zero. Methods of designing PI controller for such unstable systems without any zero, are available.[1–6] The response shows a large overshoot when compared to that of stable systems. Methods of designing PI/PID controllers for stable systems with an unstable zero, are available.[7–11] The closed loop response of such a system shows a large initial inverse response. One of the simple methods of designing PI/PID controller is the synthesis method.[12–13] For unstable systems, the synthesis method is reported for systems without zero and without delay.[13] Presence of an unstable zero, particularly dominant zero, the unstable zero is nearer to the imaginary axis (in the S-plane) than that of the unstable pole, it poses difficulty in designing a controller. Such a transfer function model occurs in the modeling of an isothermal continuous stirred tank reactor for carrying out an enzymatic reaction[14]. Due to the measurement delay, a time delay occurs in the transfer function model. Method of designing controllers for such system is not available. In the present work, controllers for the system $k_p\,(1\text{-}ps)/(\tau s\text{-}1)$ without and with time delay, are designed by the synthesis method. A set point weighted controller is proposed to reduce the initial jump and the undershoot in the response.

Model equation for bioreactor

A schematic diagram of the reactor is shown in Fig 1. We consider an isothermal, continuously stirred tank reactor for carrying out an enzymatic reaction with the reaction rate given by $-k_1\,c/(1+k_2c)^2$. This particular rate form has been extensively studied and its applicability to enzyme catalyzed reactions has been demonstrated[15–16]. The non ideal mixing is described by Cholette's model[17]. In Fig.1, n is the fraction of the reactant feed that enters the zone of the perfect mixing and m is the fraction of the total volume of the reactor where the reaction occurs [i.e., (1-m) fraction of the volume is dead zone]. The transient equation of the reactor[14] is given by

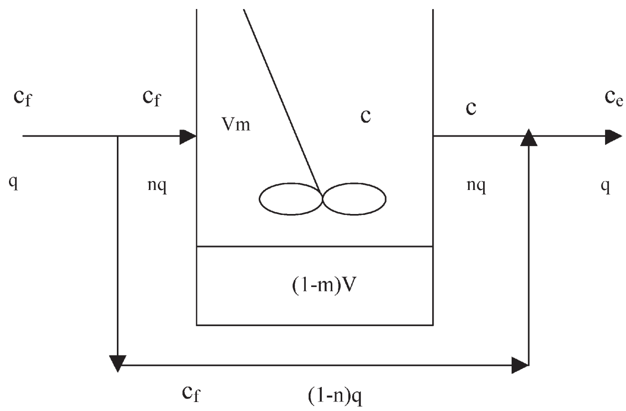

Fig. 1 – *Schematic representation of the Cholette's model of a continuously mixed enzymatic reactor*

$$\mathrm{d}c/\mathrm{d}t = [nq/(mV)]\,(c_f - c) - [k_1 c/(1+k_2 c)^2] \quad (1)$$

$$\text{at } t = 0,\ c = c_0 \quad (2)$$

Here c (concentration in the well mixed zone) and c_e (reactor exit concentration) are related by:

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$$nc = (1\text{-}n)\ c_f + c_e \tag{3}$$

The following values for the parameters are assumed:

$$n = 0.75, \quad m = 0.75, \quad k_1 V/\mathrm{q} = 300,$$
$$k_1 = 10\ \mathrm{s}^{-1}; \quad k_2 = 1\ \mathrm{dm}^3\ \mathrm{kmol}^{-1}; \tag{4}$$
$$V = 1\ \mathrm{dm}^3$$

The steady state analysis of the reactor model equation shows that the reactor exhibits three input and three output multiplicities.[17,14] For example, to obtain $c_e = 1.8$ kmol m^{-3}, c_f can take any of the three values: 7.024, 6.484 and 3.288 kmol m^{-3}. For designing a controller, the manipulated variable considered here is c_f and the output variable is c_e. By linearizing the nonlinear model equation, it is observed[14] that the transfer function model at c_f = 7.024 is stable. The transfer function model[14] relating the deviation in the output variable (c_e) and that in the manipulated variable (c_f) at the operating condition c_e = 1.8 and c_f = 6.484 is of the form k_p $(1\text{-}ps)/(\tau s\text{-}1)$ with $p > \tau$ and p = 4.473 and τ = 3.1 and k_p = –0.1727. As stated earlier, the design and analysis of controllers for such systems are not available.

Controller Design for system without delay

Let us first consider the reactor without any measurement delay. The transfer function model considered here is of the form

$$G_p = y/u = k_p\ (1-ps)/(\tau s-1) \tag{5}$$

The closed loop transfer function for a servo problem is given by

$$y/y_r = G_c\ G_p\ /[1+G_c\ G_p] \tag{6}$$

Here y is the deviation variable in c_e from its nominal value and u is the deviation variable in c_f from its nominal value. Here y_r is the set point value for y. The transfer function of the controller is obtained from Eq(6) as:

$$G_c = (y/y_r)/[\{1 - (y/y_r)\}G_p] \tag{7}$$

Let us assume the closed loop transfer function as

$$y/y_r = (1-ps)\ (1+\eta s)/[(1+\tau_1 s)(1+\tau_2 s)] \tag{8}$$

It is easier to specify the desired closed loop time constants τ_1 and τ_2 based on the knowledge of the open loop time constant (τ). Method of obtaining expression for η will be given later. Substituting Eq(8) and Eq(5) in Eq(7) gives

$$G_c = (\tau s - 1)(1+\eta s)/$$
$$[\{(\tau_1 s+1)(\tau_2 s+1) - (1-ps)(1+\eta s)\}k_p]. \tag{9}$$

The denominator (denoted by D_1) of Eq(9) is rewritten as:

$$D_1 = s(b-\eta)\ k_p\,[1 + s\{(\tau_1\tau_2 + \eta p)/(b-\eta)\}] \tag{10}$$

where

$$b = \tau_1 + \tau_2 + p \tag{11}$$

Let

$$(\tau_1\tau_2 + \eta p)/(b-\eta) = -\tau \tag{12}$$

Then from Eq(10) and the denominator of Eq(9) one gets:

$$(1+\tau_1 s)\ (1+\tau_2 s) - (1-ps)(1+\eta s) =$$
$$-\ s(b-\eta)(\tau s - 1) \tag{13}$$

Hence, on substituting the expression in the right side of Eq(13) for the corresponding denominator term in Eq(9), we get a PI controller:

$$G_c = k_c\ [1 + (1/\tau_I s)] \tag{14}$$

where

$$k_c = \eta/[(-b+\eta)k_p] \tag{15}$$

$$\tau_I = \eta \tag{16}$$

From Eq(13), we get the expression for η as

$$\eta = [\tau_1\tau_2 + (\tau_1 + \tau_2)\,\tau + \tau p]/(\tau - p) \tag{17}$$

Controller design for the system with delay

Let the process transfer function be given by

$$G_p = k_p\ (1-ps)\ \exp(-Ls)/(\tau s - 1) \tag{18}$$

In the synthesis method, the desired closed loop transfer function is to be specified. Since the process transfer function has a positive zero and time delay, they will appear in the closed loop transfer function. Usually the closed loop system has two dominant poles. Therefore the closed loop transfer function is selected as:

$$y/y_r = (1-ps)(1+\eta s)\ \exp(-Ls)/[(1+\tau_1 s)(1+\tau_2 s)] \tag{19}$$

Hence, the transfer function of the controller is obtained as:

$$G_c = (\tau s - 1)(1+\eta s)\ \exp(-\mathrm{L}s)/$$
$$[\{(\tau_1 s+1)(\tau_2 s+1) - (\eta s+1)(1-ps)\ \exp(-Ls)\}k_p] \tag{20}$$

Using the approximation $\exp(-Ls) = 1 - Ls$, we can write the denominator (denoted by D_2) of Eq(20) as:

$$D_2 = -a_3\, s(Q_1 s^2 + Q_2 s - 1)\, k_p \quad (21)$$

where

$$a_3 = (h_1 - \eta) \quad (22)$$

$$h_1 = (\tau_1 + \tau_2 + p + L) \quad (23)$$

$$Q_1 = pL\, \eta / a_3 \quad (24)$$

$$Q_2 = -[\tau_1\, \tau_2 + (p + L)\, \eta - pL]/a_3 \quad (25)$$

Let

$$D_2 = -k_p\, a_3\, s(\alpha s + 1)\, (\tau s - 1) \quad (26)$$

Then Eq(20) becomes

$$G_c = [1/(\alpha s + 1)]\, k_c\, [1 + (1/\tau_I s)] \quad (27)$$

where

$$k_c = -\eta/(a_3 k_p) \quad (28)$$

$$\tau_I = \eta \quad (29)$$

The controller is a PI control with a first order filter.

The expression for η and α are obtained by equating Eq(21) and Eq(26) as:

$$\eta = \tau(\tau_1\tau_2 - pL + h_1\tau) \,/\, [\tau^2 + pL - (p+L)\tau] \quad (30)$$

$$\alpha = pL\eta \,/\, [(h_1 - \eta)\, \tau] \quad (31)$$

Set point weighted PI controller

Systems with an unstable pole give a large overshoot.[1] For stable systems, the use of set point weighting parameter is suggested to reduce the overshoot.[18,19,20] Stable system with an unstable zero gives a large initial inverse response.[9] In the present work, a method for calculating the set point weighting parameter for an unstable system with an unstable zero is proposed by extending the method suggested by *Chidambaram*.[19,20] The PI controller law with the set point weighting parameter is given by

$$u(t) = k_c\, [(\beta y_r - y) + (1/\tau_I s) \int e\, dt] \quad (32)$$

where u is the controller output and β is the set point weighting parameter. For systems without any zero, *Chidambaram*[19] has derived the equation for β as

$$\beta\tau_I = \tau_e\zeta \quad (33)$$

where ζ is the damping coefficient and τ_e is the reciprocal of the natural frequency of the closed loop system. For the system without any zero, the numerator of the closed loop system contains only the term $(\beta\tau_I s + 1)$. Whereas for system with a zero, the numerator of the closed loop system becomes $(\beta\tau_I s + 1)(1 - ps)$. The two terms can be combined as $[(\beta\tau_I - p)s + 1]$. It can be easily shown from the work of *Chidambaram*[19] that the following equation holds good:

$$(\beta\tau_I - p) = \tau_e\zeta \quad (33)$$

Hence, the value of the set point weighting parameter is calculated as:

$$\beta = (p + \tau_e\zeta) \,/\, \tau_I \quad (34)$$

For the calculation of β, we need to calculate the closed loop ζ and τ_e. Since, for the synthesis method, we specify the closed loop transfer function by Eq(19), we get $\zeta = (\tau_1 + \tau_2)/2\tau_e$ and $\tau_e = (\tau_1\, \tau_2)^{0.5}$. Hence, it is easier to calculate the value for β by Eq(34).

Simulation results

Let us first consider the unstable system given by Eq(5) without any time delay. For the transfer function model quantities ($p = 4.473$, $\tau = 3.1$ and $k_p = -0.1727$), we get the controller settings as: $k_c = -4.8615$ and $\tau_I = -128.08$. The values of $\tau_1 = \tau_2 = 10$ are assumed. The PI controller has a negative value for the integral time. It should be noted that in case the system has a positive non-dominant zero (i.e., when compared to the zero location, the pole should be nearer to the imaginary axis), the method then leads to a conventional PID controller with a stable first order filter. The performance of the controller is evaluated on the nonlinear model equations with the nominal operating conditions: $c_f = 6.484$ and $c_e = 1.8$. The response in c_e for a step change (0.1) in the set point is evaluated and the response is shown in Fig 2. The manipulated variable behaviour is shown in Fig 3. The response shows a larger jump in the response[c_e (at $t = 0$) = 2.364]. The response reaches the desired value of 1.9 after an undershoot of 0.267. The effect of τ_1 (= τ_2) is also studied for three values of τ_1 as 10, 15 & 20. Fig. 2 shows the response for $\tau_1 = 10$ is preferred. Since the integral time is obtained as negative, let us check whether the closed loop is stable for perturbation in the integral time. The value of the integral time is varied by + 10 % of the actual value and separately by –10 % of the actual value. Fig 4 shows that the system is stabilized. Appendix-A gives the details on the proof that the system is stabilized for perturbation in the gain, the time constant(k_p, p and τ) of the system. The regulatory response is evaluated for a step change in the distur-

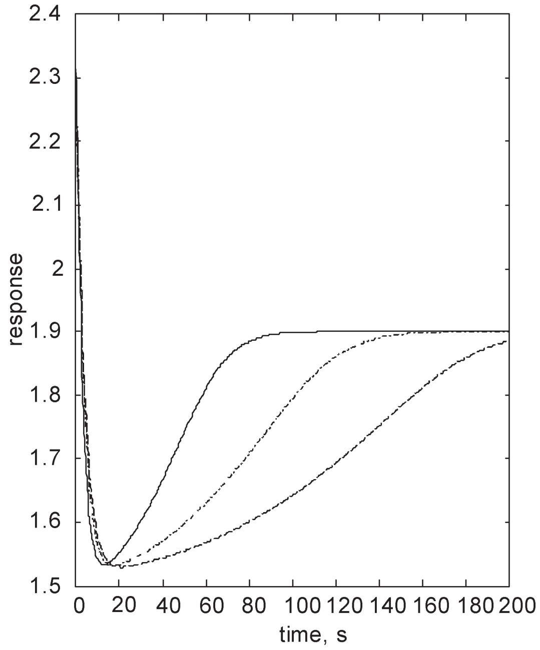

Fig. 2 – *The servo response for a step change of 0.1 solid: $\tau_1 = \tau_2 = 10$; dot $= \tau_1 = \tau_2 = 15$; dash $= \tau_1 = \tau_2 = 20$*

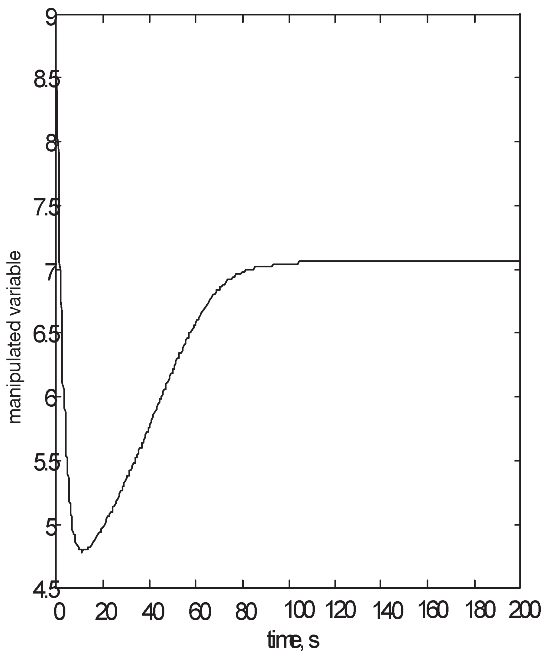

Fig. 3 – *Manipulated variable versus time behaviour $\tau_1 = \tau_2 = 10$*

bance variable (q) from 0.03666 x 10^{-3} to 03666 × 10^{-3} m^3 s^{-1}. The response is shown in Fig. 5. The system is stabilized.

Since the initial jump in the response is large, the set point weighted PI controller can be considered. From Eq(34), we get the value of β as –0.113. The value of β is obtained as negative value. The use of $\beta = 0$, eliminates zero in the closed loop re-

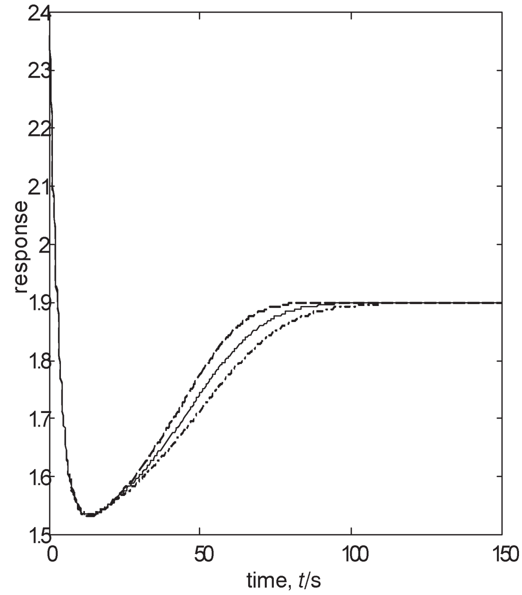

Fig. 4 – *The effect of perturbation in integral time on the servo response solid: $\tau_I = -128.08$, dot: $\tau_I = -149.89$ (+ 10 % perturbation), dash: $\tau_I = -115.27$ (–10 % perturbation)*

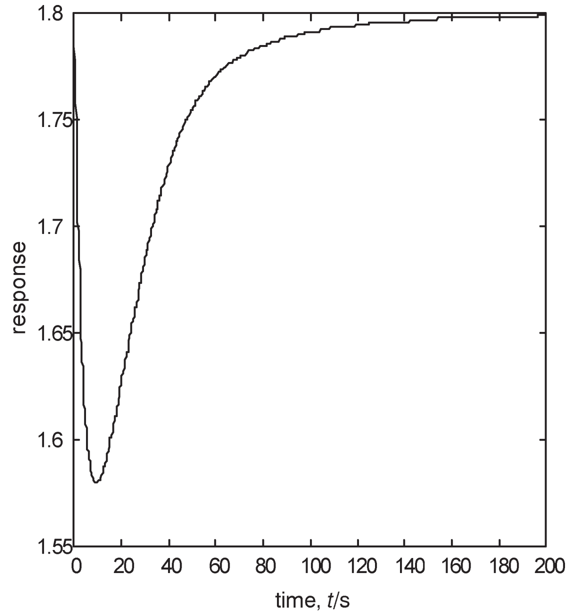

Fig. 5 – *Regulatory response of the reactor for a step change in q from 0.03333 × 10^{-3} m^3 s^{-1} to 0.03666 × 10^{-3} m^3 s^{-1}*

sponse. Fig 6 shows the set point weighted PI controller performance. Fig 6 shows that the initial jump is significantly reduced. Fig. 6 shows that $\beta = 0$ makes the response sluggish. Hence, the derived optimal value for β is to be used.

Let us consider the stabilization of the system with a measurement delay of 0.1 s. The synthesis method gives (for assumed value of $\tau_1 = \tau_2 = 30$) a PI controller with a first order filter. $k_c = -5.3713$

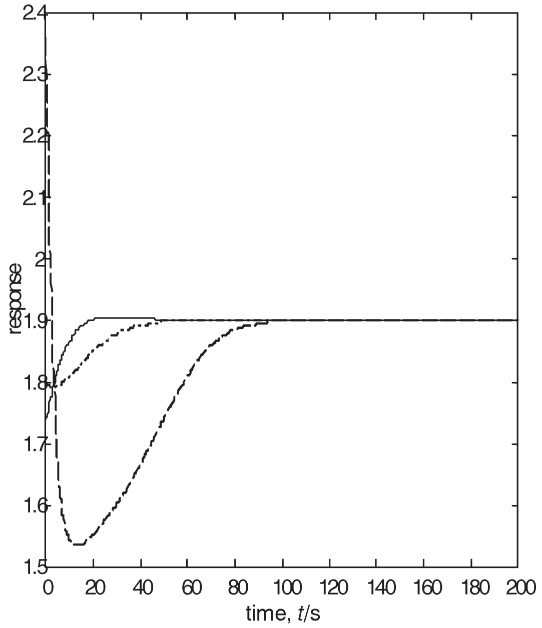

Fig. 6 – *Effect of set point weighting parameter on the servo response*
dot: β = 1, dash: β = 0, solid: β = –0.113

and τ_I = –827.6068 and α = –0.1339. Here for the dominant zero unstable system, both the integral controller time and the filter time constant are negative. That is, the controller is an unstable one. Thus, the presence of measurement delay poses restriction on the design of controllers. Appendix-B gives the condition under which the system could be stabilized. Fig. 7 shows the servo response of the system for a step change in the set point. The performance of the reactor without and with set point weighting

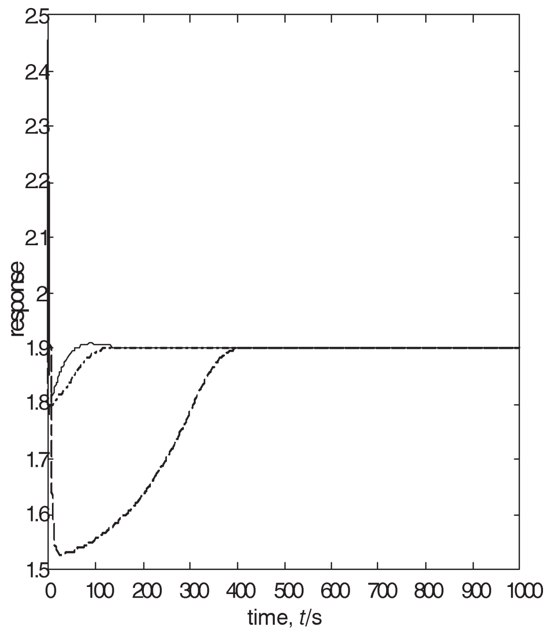

Fig. 7 – *The servo response of the reactor system with measurement delay*
dot: β = 1, dash: β = 0, solid: β = –0.0417

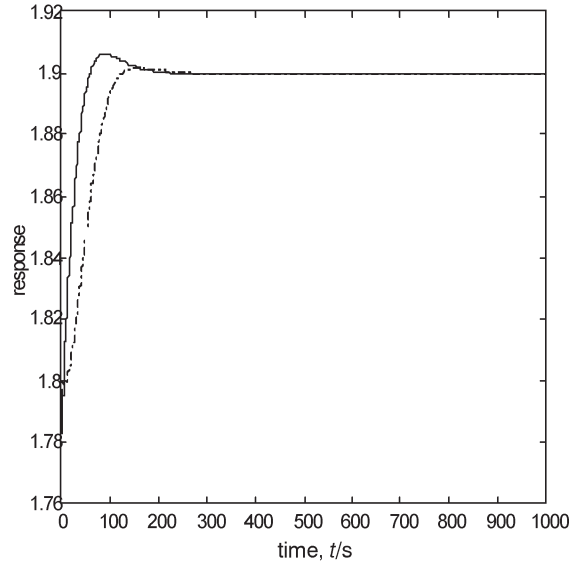

Fig. 8 – *Comparison of the servo response for β = 0 and β = –0.0417*

parameter, is also shown in Fig. 7. The set point weighting drastically reduce the initial jump (from 2.46 to 1.77) and the undershoot (from 0.27 to 0.0). Fig. 8 shows the servo responses for the two set point weighting quantities β = 0 and β = – 0.0417. The response is faster for β = – 0.0417. The regulatory response for a step change in q = 0.03333x10^{-3} to 0.03666x10^{-3} m^3 s^{-1} is shown in Fig 9. The reactor is stabilized. In Appendix-B, the stability condition is derived (for dominant unstable zero condition) as [-(L/p) + (L/τ)] < 0.62. It has to be noted that when the transfer function has an unstable pole and non dominant unstable zero, it is found that the synthesis method gives positive values for τ_I and α. That is, the controller is a stable one. Since the open loop system is unstable, it has to be always kept in

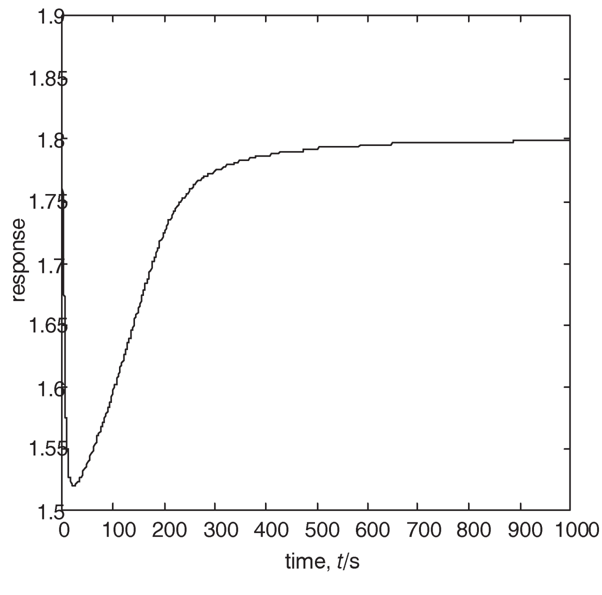

Fig. 9 – *Regulatory response for the reactor with delay*

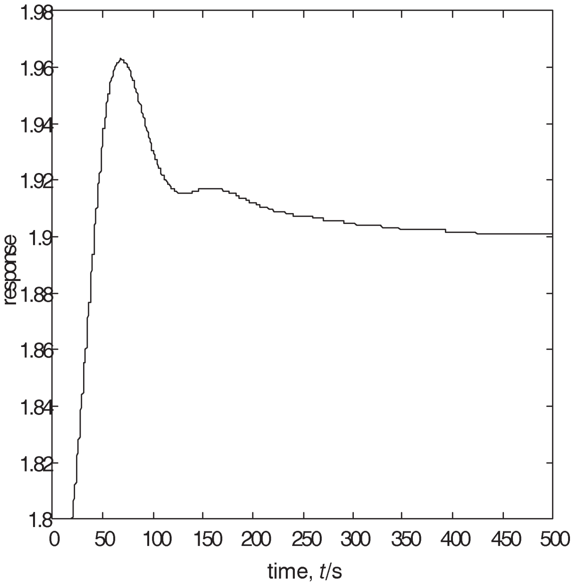

Fig. 10 – *The servo response for the operating condition c_f =3.288 kmole/m^3*

closed loop with a controller to stabilize the system. It has to be noted that when the operating condition for the bioreactor is at c_f = 3.288, then the transfer function has an unstable pole and a stable zero: k_p = 2.2078, τ = 98.32 , p = –11.133. The synthesis method (with $\tau_1 = \tau_2 = 50$) gives the controller settings as k_c = 1.5456, τ_I = 153.99 and α = 7.728. The controller is a stable one. Fig. 10 shows the servo response of the reactor.

Conclusions

For stabilizing the bioreactor for carrying out an enzymatic reaction, controllers are designed by the synthesis method. Since, the transfer function model has an unstable pole and a dominant unstable zero, the synthesis method gives a PI controller with a negative integral time. When a measurement delay is introduced, the controller is found to be a PI controller with an unstable first order filter. Stability analysis for the controller requires the condition $[(L/\tau) - (L/p)] < 0.62$. Simulation results on the nonlinear model equations are evaluated for both the servo and regulatory responses. The proposed controller stabilizes the nonlinear reactor. The proposed set point weighting reduces significantly the initial jump and the undershoot of the servo response.

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Appendix – A:

Robustness of PI controller with negative integral time

Let the controller is designed for the process k_p, τ and p whereas the system is simulated on the process with parameters k_p', τ' and p'. Hence the characteristic equation is given by

$$a_1 s^2 + a_2 s + k_p' = 0 \qquad \text{(A.1)}$$

where

$$a_1 = [\tau_1\tau_2(k_p\tau' - k_p'p') + b(k_p\tau'p - k_p'\tau p')]/(\tau - p) \qquad \text{(A.2)}$$

$$a_2 = [\tau_1\tau_2(k_p' - k_p) + b(k_p'\tau - pk_p) - k_p'p'(\tau - p)]/(\tau - p) \qquad \text{(A.3)}$$

We have to show that a_1 and a_2, both, should be positive for perturbation in the model parameters k_p, τ and L.

Let us consider that there is positive change in p in the process (i..e., $p' > p$). It can be easily checked that both a_1 and a_2 are positive. It should be noted that b can be selected by increased values of closed loop time constants. Similarly, we can also show that both a_1 and a_2 are positive for positive perturbation in k_p. It is also verified by simulation study that for, both, the increased and decreased perturbations of the model parameters (+ 10 % or –10 %) that all the coefficients (a_1 and a_2) are found to be positive.

Appendix-B:

Assessment of the stabilizing controller

In this section, a condition is derived for stabilization of an unstable system with dominant unstable zero. *Ho* and *Xu*[21] have proposed a method to assess the control system for an unstable system. They have considered unstable systems without any zero. In this section, their work is extended to systems with an unstable zero.

The open loop transfer function of the system with a PI controller and a filter is given by

$$kG = k_c k_p (1-ps)[(\tau_I s-1)/(\tau_I s)] \exp(-Ls)/[(1-\alpha s)(\tau s-1)] \quad \text{(B.1)}$$

The gain margin conditions are given by

$$0.5\pi+\tan^{-1}(\tau\omega_p) - \tan^{-1}(\tau_I\omega_p) - \tan^{-1}(p\omega_p) + \tan^{-1}(\alpha\omega_p) - L\omega_p = 0 \quad \text{(B.2)}$$

$$A_m k_c k_p = \omega_p \tau_I \{[(1+\tau^2\omega_p^2)(1+\alpha^2\omega_p^2)]/[(1+\tau_I^2\omega_p^2)(1+p^2\omega_p^2)]\}^{0.5} \quad \text{(B.3)}$$

where A_m is the gain margin.

Similarly the phase margin criteria is given by

$$0.5\pi + \tan^{-1}(\tau\omega_g) - \tan^{-1}(\tau_I\,\omega_g) - \tan^{-1}(p\omega_g) + \tan^{-1}(\alpha\omega) - L\omega_p = \phi_m \quad \text{(B.4)}$$

$$k_c k_p = \omega_g\,\tau_I \{[(1+\tau^2\omega_g^{\;2})(1+\alpha^2\omega_g^{\;2})/[(1+\tau_I^2\omega_g^{\;2})(1+p^2\omega_g^{\;2})]\}^{0.5} \quad \text{(B.5)}$$

where ϕ_m is the phase margin.

Since the argument in $\tan^{-1}(.)$ is always found to be greater than 1, then we can use the approximation:

$$\tan^{-1}(x) = 0.5\pi - (1/x) \text{ for } x > 1 \quad \text{(B.6)}$$

Under the conditions of larger argument for tan(.), Eqs (B.3) and (B.5) become

$$A_m\,k_c k_p = \omega_p\,\tau\,\alpha/p \quad \text{(B.7)}$$

$$k_c k_p = \omega_g\,\tau\alpha/p \quad \text{(B.8)}$$

Hence we get,

$$A_m = \omega_p/\omega_g \quad \text{(B.9)}$$

Using Eq(B.6) in Eq(B.2) & (B.4) we get

$$0.5\pi + (1/\omega_p)\,[(1/\tau_I) + (1/p) - (1/\alpha) - (1/\tau)] - L\omega_p = 0 \quad \text{(B.10)}$$

$$\phi_m = 0.5\pi + (1/\omega_g)\,[(1/\tau_I) + (1/p) - (1/\alpha) - (1/\tau)] - L\omega_g \quad \text{(B.11)}$$

From Eqs (B.10), (B.9) and B(11), we get

$$[\phi_m\,A_m + 0.5\pi\,A_m(A_m\text{-}1)\,]/[(A_m^{\;2}\text{-}1)L] = \omega_p \quad \text{(B.12)}$$

From Eq(B.10) alone we get

$$[(1/\alpha) - (1/\tau_I)\,] = (1/p) + (1/\tau) - 0.5\pi\,\omega_p + L\,\omega_p^{\;2} \quad \text{(B.13)}$$

The first condition to be imposed is $k_c\,k_p > 0$. This from Eq(B.7) leads to the condition

$$\omega_p > 0 \quad \text{(B.14)}$$

Hence Eq(B.12) becomes

$$[\phi_m\,A_m + 0.5\pi\,A_m(A_m - 1)\,]/[A_m^{\;2} - 1)L] > 0 \quad \text{(B.15)}$$

Imposing the second condition that the left side of Eq (B.13) should be greater than zero (because the filter time constant is less than that of the integral time):

$$\omega_p > h_2/L \quad \text{(B.16)}$$

where

$$h_2 = 0.25\,\pi + 0.5\,\{(0.25\,\pi^2 + 4\,[(-L/p) + (L/\tau)]\}^{0.5} \quad \text{(B.17)}$$

From Eq(B.12) and Eq(B.16) we get

$$\phi_m = [h_2\,(A_m^{\;2} - 1)/A_m] - [0.5\pi(A_m - 1)] \quad \text{(B.18)}$$

To maximize the phase margin, we need $d\phi_m/dA_m = 0$ (B.19)

For Eq (B.18), this condition leads to

$$\phi_{m,\,max} = -2\,\{[(-L/p) + (L/\tau)]\}^{0.5} + 0.5\pi \quad \text{(B.20)}$$

By imposing the condition, this maximum value of phase margin should be greater than zero, we get the condition under which the system can be stabilized:

$$[-(L/p) + (L/\tau)] < 0.62$$

Nomenclature

A_m – amplitude ratio
a_1, a_2 – defined by Eq(A.1) and Eq(A.2)
a_3 – defined by Eq(22)
b – defined by Eq(11)
c – concentration in the well mixed zone, k mol^{-3}
c_e – exit concentration of the reactor, k mol^{-3}
c_f – reactant concentration in the feed, k mol^{-3}
D_1, D_2 – defined by Eq(10) and Eq(21) respectively
e – error defined by ($y_r - y$)
G_p, G_c – transfer function of the process and the controller respectively
h_1 – defined by Eq(23)
h_2 – defined by Eq(B.17)
k_1 – reaction rate constant, s^{-1}
k_2 – inhibition constant, k mol^{-3}
k_p, k_c – gain of the process and the controller respectively
L – measurement delay, s
m – fraction of total volume of reactor where the reaction occurs
n – fraction of the feed entering the reaction zone
p – time constant of the numerator dynamics
q – volume flow rate to the reactor, m^3 s^{-1}
Q_1, Q_2 – defined by Eq(24) and Eq(25) respectively
u – manipulated variable
V – volume of the reactor, m^3
y – output variable to be controlled
y_r – set point value for y
α – filter time constant of the controller defined by Eq (31), s
β – set-point weighting parameter
τ_1,τ_2 – closed loop time constants, s
τ – process time constant, s
τ_e – effective time constant of the closed loop system, s
τ_I – integral time, s
ζ – damping coefficient
ϕ_m – phase margin
$\phi_{m,max}$ – maximum phase margin
ω_p, ω_g – phase cross over frequency and gain cross over frequency respectively
η – defined by Eq(17) & Eq(30) respectively for system without & with delay