Haar Orthogonal Functions Based Parameter Identification of Wastewater Treatment Process with Distributed Parameters

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The most often used system in aerobic biological wastewater treatment is the system "biological reservoir – sedimentor". The necessary condition for obtaining a good operative control is an adequate process model to be available. The aim of this paper is modelling of a process, carrying out a system "biological reservoir – sedimentor", and consequent parameter identification. In order to obtain a model with higher degree of accuracy, the process of wastewater treatment is considered as object with distributed parameters. Shifted two – dimensional Haar orthogonal functions are used for parameter identification of the process. The implementation of these orthogonal functions reduces the problem to a computationally convenient form. The algorithm is efficient and simple in form.

Key words:
Modelling, identification, Haar orthogonal functions, wastewater treatment, distributed parameters objects.

Introduction

The methods of biological wastewater treatment lie at the root of the most of the conventional operative wastewater treatment stations. In the practice of aerobic biological treatment of wastewater with organic contamination, the system "biological reservoir (or bioreservoir) – sedimentor" (SBS) is the most often used. In order to achieve a good operative control of SBS, the most important task is an adequate mathematical model of process dynamics to be elaborated. The model has described the basic biochemical and physicochemical processes in the aerobic biological treatment. There is a wide variety of mathematical models of aerobic biological treatment processes, among that diffusion, sorption, kinetic, deterministic, stochastic, as well as a lot of empiric models. But there are a comparatively small number of mathematical models, obtained at dynamical running of SBS processes. In the last few years there is a high scientific interest in the field of simulation investigations of wastewater treatment processes from the control point of view.[1–4] Despite of number of positive properties, the practical applications of dynamical modelling of wastewater treatment processes is rather limited.[3,5] In order to overcome some difficulties when modelling this kind of processes, some new methods have been elaborated.[6] It is caused by the fact that nowadays it is quite impossible some experimental data to be interpreted without application of mathematical models.

When biotechnological processes, and particularly wastewater treatment processes, have to be modeled, a lot of complicated reactions and interdependent characteristics have to be described. If an assumption of uniform distribution of the bioreservoir components is accepted, these processes have been considered as objects with fixed parameters. This assumption has a meaning if there is a small bioreactor or almost complete stirring, but neither in big volumes, such as bioreservoirs. The consideration of the space distribution of the characterized process variables leads to the description of wastewater treatment processes as objects with distributed parameters. This is a preposition for obtaining a more precise process model. Partial differential equations or systems of partial differential equations are often used for the processes behavioral description in the class of objects with distributed parameters.

In order to prove the validity of the model, parameter estimation of process parameters has to be done, based on the experimental data from operative wastewater treatment station. In the previous authors' works the conventional optimization pro-

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cedures, like a *Simplex Search* or *Minimax*, have been used,[7] as well as block impulse functions.[8]

Orthogonal functions method is often used to solve the parameter identification problem of distributed systems. The method is based on expansion of the functions of equation, expressing the system into orthogonal polynomials or functions.

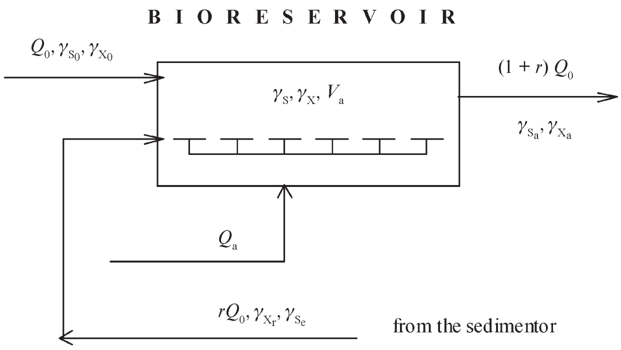
Shifted orthogonal Laguerre, Legendre and Chebyshev polynomials; Walsh, Haar, block – pulse, and sine – cosine functions are implemented for approximation of table data and analytically given functions by *Genov* and *Todorova*.[9] It should be noticed that the obtained approximation accuracy of different signals is bigger when the Haar orthogonal functions are implemented. Except these functions there are the simplest wavelet functions and they have interesting features – computational attraction and low computer memory requirement. So, the attention is turning to Haar wavelet functions. *Chen* and *Hsiao*[10] established in 1996 an operational matrix of integration based on Haar wavelets, and formulated a procedure for applying the matrix to analyze lumped and distributed parameter's dynamic systems. Later *Todorova* and *Genov* develop the method of Haar wavelets for estimating the parameters, initial and boundary conditions of classes of distributed parameter systems.[11–15]

The aim of this paper is an adequate mathematical model of the aerobic biological treatment processes in SBS, described in the class of distributed parameters objects, to be obtained, and the application of shifted two – dimensional Haar wavelets for parameter identification to be presented.

Description and modelling of wastewater treatment processes in the system "bioreservoir – sedimentor"

In contrast to the conventional bioreactors for cultivation of microorganisms, where the basic aim is to produce a maximum amount of biomass, here, in the system "bioreservoir – sedimentor", the basic aim is some low concentration of organic contaminating substances in sedimentor outlet to be kept. For the correct running of aerobic biological treatment in the SBS, it is necessary to have a good observation and control of both biochemical processes in the bioreservoir, as well as of the mechanic processes in secondary sedimentor. If one of two devices does not work in an optimal way, it could turn down the whole process of aerobic biological treatment.

The most of known mathematical models of the SBS have been developed on the basis of the classical technological scheme of SBS, fully presented in.[16] The mathematical models, presented also in[16] are developed on the assumption, that the decrease of organic contamination as well as increase of active sludge, carry out only in the bioreservoir. Due to the fact, that the basic biochemical processes run only in the bioreservoir, while the processes in the sedimentor are mainly mechanical, it is of major interest the concentration variations of organic contamination (substrate) and active sludge (biomass) in the bioreservoir to be considered. Therefore, the basic aim of consideration in this paper will be only the processes carried out in the bioreservoir. Due to this, the complete technological scheme, presented in,[16] is reduced only to the bioreservoir and the inflows and outflows. The reduced scheme of the bioreservoir is presented in Fig. 1.



γ_S, γ_X – concentrations, respectively, of organic contaminations (substrate) and active sludge (biomass) in the bioreservoir, g l^{-1}; Q – capacity of wastewater, l h^{-1}; Q_a – capacity of inflow air, l h^{-1}; subscripts *o*, *a*, *e* – denote inlet, outlet and recirculation concentrations; *r* – quotient of capacities of recirculating active sludge and inlet wastewater; V_a – useful volume of bioreservoir, l.

Fig. 1 – *Classical technological scheme of bioreservoir*

The experiments are carried out in the operative wastewater treatment station. The detailed technological scheme of this wastewater treatment station, as well as full experiment description, could be found in.[16] The system "bioreservoir – sedimentor" in this wastewater treatment station consists of six bioreservoirs, each of them with three corridors, three radial secondary sedimentors, air-pump, pump for the active sludge with a draw shaft, a distribution shaft for the serve of recirculating active sludge to the bioreservoirs, a distribution shaft for the serve of outlet suspension from the bioreservoirs to the secondary sedimentors, as well as air-mains and water-mains with the corresponding brake and control devices.

The experiments are carried out only in one bioreservoir and in one secondary sedimentor because of the fact that the entered wastewater, the recirculating active sludge, and needed air are practically equal, both, in capacity and distribution for the six bioreservoirs. In the considered bioreservoir six control points are chosen. The experimental data have been received at every two hours. The whole set of obtained experimental data are systematized, analyzed, and visualized.[16]

The following problem is formulated in this paper: the wastewater treatment process in the system "bioreservoir – sedimentor" to be considered as object with distributed parameters and a mathematical model, based on the material balance equations, is to be obtained. The model has to describe, both, in time and in geometric space, the variations of basic biochemical variables, namely substrate and biomass, and the consideration to be limited only in the bioreservoir. After the model has been developed, the parameter identification has to be presented.

Before proceeding to a process modelling in the class of distributed parameters objects, some remarks about modelling of this process in the class of fixed parameters will be shortly presented here. After the comparative analysis between specific growth rates, describing the kinetics of Monod, Contois and Andrews, the best results are obtained after application of Contois' kinetics. The parameter estimation has been fulfilled using *MATLAB Optimization Toolbox* and *Genetic Algorithms Toolbox.*[17,18] The application of genetic algorithms has been forced because of easiest and less-time consuming methods such as *Simplex Search, Minimax* etc.; they have not given satisfying results. The complete comparative analysis between the different ones mentioned above, specific growth rates of biomass, as well as the different methods for parameter estimation, could be found in.[8,19]

The description of the process variables variations, when the wastewater treatment processes are considered as object with distributed parameters, is realized in a similar way as the processes of heat distribution and diffusion processes. It is realized on the basis of diffusion equation and on the worked out of the material balance equation, which could be presented in following form:[20]

$$\frac{\partial\gamma}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(r\,\boldsymbol{u}_{\mathrm{r}}\gamma)+\frac{\partial}{\partial z}(\boldsymbol{u}_{\mathrm{z}}\gamma)=$$

$$=\frac{1}{r}\frac{\partial}{\partial z}\left(D_{\mathrm{eff}}r\frac{\partial\gamma}{\partial r}\right)+\frac{\partial}{\partial z}\left(D_{\mathrm{eff}}\frac{\partial\gamma}{\partial z}\right)+ \qquad (1)$$

$$+\ \text{reaction} + \text{mass transfer gas/liquid},$$

where:

– γ is a continuous and differentiable function, describing mass concentration;

– r, z – radial and axial co-ordinates;

– D_{eff} – turbulent dispersion coefficient;

– $\boldsymbol{u}_{\mathrm{r}}$, $\boldsymbol{u}_{\mathrm{z}}$ – radial and axial components of the rate vector.

In fact (1) presents the equation of the material balance for the concentration.

Due to the specificity of the bioreservoir, the distribution in a radial, namely direction x, is considered, while the variables do not change in an axial direction. At this stage the mass transfer from a gas to a liquid phase is not rendered. In the previous author's works[7] it is proved that the rendering of dispersion phenomena influence does not lead to more precise process model. This statement is there confirmed with graphic and numeric presentation of the considered variables (substrate and biomass) in both cases – with and without rendering of dispersion phenomena.[7] After the comparison and statistic evaluation, it is evaluated that the differences between results obtained, with and without rendering of dispersion phenomena are less than 7 % for substrate and less 15 % for biomass. At the same time, rendering of dispersion phenomena leads to more difficult mathematical description. So, without loss of generality, dispersion phenomena in the system could be regarded as negligible. When the dispersion phenomena are assumed to be negligible, the following common model for mass concentration in the radial direction is obtained from (1):

$$\frac{\partial\gamma}{\partial t}+\frac{\partial}{\partial x}(\boldsymbol{u}_{\mathrm{x}}\gamma)=\text{reaction }(\Gamma) \qquad (2)$$

When the space distribution of the substrate γ_{S} and the biomass γ_{X} are considered at this stage, the following assumptions are made:

1) the radial component of the rate vector $\boldsymbol{u}_x$ is expressed as a relation of flow rate of inlet water Q_0/dm^3 h^{-1}, constant in radial direction, and the constant bioreservoir cross-section S/m^2

$$\boldsymbol{u}_x=\frac{Q_0}{S} \qquad (3)$$

2) when the mass concentration of substrate γ_{S} is expressed, the reaction of the system in a wastewater treatment process is presented as follows[21]

$$\Gamma=-Y_{\mathrm{SX}}\cdot\mu(\gamma_{\mathrm{S}}(x,t),\,\gamma_{\mathrm{X}}(x,t))\cdot\gamma_{\mathrm{X}}(x,t), \qquad (4)$$

where:

– $\gamma_{\mathrm{S}}(x,t)$ and $\gamma_{\mathrm{X}}(x,t)$ are continuous and differentiable functions, which describe respectively the substrate and biomass concentrations, g l^{-1};

– $\mu(\gamma_{\mathrm{S}}(x,t),\,\gamma_{\mathrm{X}}(x,t))$ – a specific growth rate of biomass $\gamma_{\mathrm{X}}(x,t)$, reflected by Contois kinetics[22]:

$$\mu(\gamma_{\mathrm{S}}(x,t),\gamma_{\mathrm{X}}(x,t))=\mu_{\max}\frac{\gamma_{\mathrm{S}}(x,t)}{k_{\mathrm{S}}\cdot\gamma_{\mathrm{X}}(x,t)+\gamma_{\mathrm{S}}(x,t)},$$

where:

– μ_{max} is the maximum value of specific biomass growth rate, h^{-1}

– k_S – saturation constant, g l^{-1}

– Y_{SX} – yield coefficient.

The relation (4) yields the biochemical mechanism in the system, which is expressed as substrate decrement due to biomass accumulation in the culture medium.

3) when the mass concentration of biomass γ_X is expressed, the reaction of the system Γ in a wastewater treatment process is presented as follows

$$\Gamma = (\mu(\gamma_S(x,t), \gamma_X(x,t)) - k_d) \cdot \gamma_x(x,t) \quad (5)$$

where k_d is a death coefficient of biomass, h^{-1}. The relation (5) expresses the biomass dynamic, taking into account the biomass accumulation in the system as a result of the microorganisms' growth with the specific growth rate μ and the gradually dying out of biomass with a rate k_d.

Therefore, on the basis of (2), (3) and (4), when the dispersion phenomena are assumed to be negligible, the following model is obtained for the substrate space distribution

$$\frac{\partial \gamma_S(x,t)}{\partial t} = -\frac{Q_0}{S}\frac{\partial \gamma_S(x,t)}{\partial x} - \\ -k\mu_{max}\frac{\gamma_S(x,t)}{k_S(\gamma_X(x,t)) + \gamma_S(x,t)}\gamma_X(x,t) \quad (6)$$

By analogy, on the basis of (2), (3) and (5), when the dispersion phenomena in the bioreservoir are assumed to be negligible, the following model is obtained for the biomass space distribution

$$\frac{\partial \gamma_X(x,t)}{\partial t} = -\frac{Q_0}{S}\frac{\partial \gamma_X(x,t)}{\partial x} + \\ +\left(\mu_{max}\frac{\gamma_S(x,t)}{k_S(\gamma_X(x,t)) + \gamma_S(x,t)} - k_d\right)\gamma_X(x,t) \quad (7)$$

Then, the equations (6) and (7), describing the substrate and biomass concentrations, represent the mathematical model of the dynamics of the wastewater treatment processes in the system "bioreservoir-sedimentor", considered as object with distributed parameters.

Mathematical preliminaries

The orthogonal set of Haar functions is a group of square waves with magnitude of $\pm 2^{m/2}$ in some intervals and zeros elsewhere. Just these zeros make the Haar transform faster than other square functions such as Walsh's.[10] The first curve is 1 during the whole interval [0; 1]. It is called the scaling function. The second curve is the fundamental square wave, or the mother wavelet, which also spans the whole interval. All the other subsequent curves are generated from the second curve with two operations: translation and dilation. This orthogonal basis is not a recent invention. It is a reminiscent of the Walsh basis, in which Walsh function contains many wavelets to fill the interval [0; 1] completely, and to form a global basis. While each Haar function contains just one wavelet during some subinterval of time, and remains zero elsewhere, the Haar set forms a local basis. All the Haar wavelets are orthogonal to each other. Therefore, they form a very good transform basis. Any function, which is square integrable in the interval [0; 1], can be expanded into Haar wavelets.

Since the interval on which Haar functions are defined is not suitable for solving parameter identification problems, suitable transformation is required. The shifted Haar wavelets are defined[14] as

$$\boldsymbol{H}_m^*(t) = \boldsymbol{H}_1^*(t) \cdot \left(2^j \cdot t - \frac{k}{2^j}\right), \quad (8)$$

where:

– $\boldsymbol{H}_1^*(t)$ scaling function, pleased during the whole interval [0, T];

– $j \geq 0$; $m = 2^j + k$; $0 < k < 2^j$

They are orthogonal with respect to the mass function $\rho^*(t) = 1$ over the interval [0; T], where T is the observed interval.

A function $f(x,t)$ that is square integrable in the regions $t \in [0, T]$, $x \in [0, X]$ can be approximately expanded in a series of two-dimensional shifted Haar wavelets $\boldsymbol{H}_{ij}^*(x,t)$ as

$$f(x,t) \cong \sum_{i=1}^{m}\sum_{j=1}^{m} c_{ij}^* \boldsymbol{H}_{ij}^*(x,t) = \\ = \sum_{i=1}^{m}\sum_{j=1}^{m} c_{ij}^* \boldsymbol{H}_i^*(t) \cdot \boldsymbol{H}_j^*(x) = \\ = (\boldsymbol{H}_M^*(t))^T \cdot \boldsymbol{C}_M^* \cdot \boldsymbol{H}_M^*(x), \quad (9)$$

where:

$\boldsymbol{C}_M^*$ is $(m \times m)$ matrix with expansion coefficients c_{ij}^*;

$\boldsymbol{H}_M^*(t) = [\boldsymbol{H}_1^*(t) \cdot \boldsymbol{H}_2^*(t) \ldots \boldsymbol{H}_m^*(t)]^T$;

$\boldsymbol{H}_M^*(x) = [\boldsymbol{H}_1^*(x) \cdot \boldsymbol{H}_2^*(x) \ldots \boldsymbol{H}_m^*(x)]^T$;

Parameter identification process

The Haar wavelets implementation reduces the problem of parameter identification of the processes in distributed parameter objects to a computationally convenient form. The partial differential equations (PDE), expressing the object, are transformed into set of algebraic equations, and the algorithm for estimating of the parameters of the object can be derived in a discrete form. Compared with the classical methods, the Haar method is computationally simplest, faster, and has low computer memory requirement.[14]

The identification process includes the following fundamental steps:

(i) expansion of the functions of PDE into shifted two-dimensional Haar wavelets;

(ii) rewriting of the PDE in the matrix form using the Haar wavelets properties, and after some well known manipulations;[10,11,12,14]

(iii) solving of the obtained matrix equation for the vector of unknown parameters using least – squares technique.

Before applying the identification algorithm, the system of partial differential equations (6) and (7), describing the processes in SBS, is presented in the form

$$\left|\begin{array}{l} k_S a_{11}(x,t) + Y_{SX}\mu_{\max} a_{12}(x,t) = h_1(x,t) \\ k_S a_{21}(x,t) + (k_d - \mu_{\max}) a_{12}(x,t) + \\ \qquad + k_d k_S a_{23}(x,t) = h_2(x,t) \end{array}\right. , \quad (10)$$

where:

$$a_{11}(x,t) = \left(\mathrm{XIS}(x,t) \frac{\partial \gamma_S(x,t)}{\partial t} - u_{XX}(x,t) \frac{\partial \gamma_S(x,t)}{\partial x} \right);$$

$$a_{12}(x,t) = \gamma_S(x,t) \cdot \gamma_X(x,t);$$

$$h_1(x,t) = -u_{XS}(x,t) \frac{\partial \gamma_S(x,t)}{\partial x};$$

$$a_{21}(x,t) = \left(\mathrm{XIS}(x,t) \frac{\partial \gamma_X(x,t)}{\partial t} - u_{XX}(x,t) \frac{\partial \gamma_X(x,t)}{\partial x} \right);$$

$$a_{23}(x,t) = \gamma_X(x,t)^2;$$

$$h_2(x,t) = -u_{XS}(x,t) \frac{\partial \gamma_X(x,t)}{\partial x};$$

$$\mathrm{XIS}(x,t) = \gamma_X(x,t) + \gamma_S(x,t);$$

$$u_{XX}(x,t) = u_X(t) \cdot \gamma_X(x,t);$$

$$u_{XS}(x,t) = u_X(t) \cdot \gamma_S(x,t);$$

$$u_X(t) = -\frac{Q_0(t)}{S}.$$

The functions of (10) are expanding into shifted two-dimensional Haar wavelets through (9):

$$a_{11}(x,t) \cong (\boldsymbol{H}_M^*(t))^T \cdot F_{11} \cdot \boldsymbol{H}_M^*(x); \quad (11)$$

$$a_{12}(x,t) \cong (\boldsymbol{H}_M^*(t))^T \cdot F_{12} \cdot \boldsymbol{H}_M^*(x); \quad (12)$$

$$a_{21}(x,t) \cong (\boldsymbol{H}_M^*(t))^T \cdot F_{21} \cdot \boldsymbol{H}_M^*(x); \quad (13)$$

$$a_{23}(x,t) \cong (\boldsymbol{H}_M^*(t))^T \cdot F_{23} \cdot \boldsymbol{H}_M^*(x); \quad (14)$$

$$h_1(x,t) \cong (\boldsymbol{H}_M^*(t))^T \cdot D_1 \cdot \boldsymbol{H}_M^*(x); \quad (15)$$

$$h_2(x,t) \cong (\boldsymbol{H}_M^*(t))^T \cdot D_2 \cdot \boldsymbol{H}_M^*(x), \quad (16)$$

where F_{11}, F_{12}, F_{21}, F_{23}, D_1 and D_2 are $(m \times m)$ shifted two-dimensional Haar wavelets coefficient matrixes of the functions of the system equations (10).

Substituting expansion above into (10), and after some manipulation is obtained

$$\left|\begin{array}{l} (\boldsymbol{H}_M^*(t))^T [q_1 F_{11} + q_2 F_{12}] \cdot \boldsymbol{H}_M^*(x) = \\ \qquad = (\boldsymbol{H}_M^*(t))^T D_1 \cdot \boldsymbol{H}_M^*(x) \\ (\boldsymbol{H}_M^*(t))^T [q_3 F_{21} + q_4 F_{12} + q_5 F_{23}] \cdot \boldsymbol{H}_M^*(x) = \\ \qquad = (\boldsymbol{H}_M^*(t))^T D_2 \cdot \boldsymbol{H}_M^*(x) \end{array}\right. , \quad (17)$$

where:

$$q_1 = \hat{k}_S; \quad q_2 = \hat{Y}_{SX} \cdot \hat{\mu}_{\max}; \quad q_3 = \hat{k}_S;$$
$$q_4 = \hat{k}_d - \hat{\mu}_{\max}; \quad q_5 = \hat{k}_d \cdot \hat{k}_s.$$

Equating the coefficients of like Haar function products in system (17) and rewriting it in matrix form gives

$$F \cdot \theta = D, \quad (18)$$

where:

$$F = \begin{bmatrix} F_1 & O_2 \\ O_1 & F_2 \end{bmatrix}_{(2m^2 \times 5)};$$

$$F_1 = \begin{bmatrix} (F_{11})_1 & (F_{12})_1 \\ \dots & \dots \\ (F_{11})_j & (F_{12})_j \\ \dots & \dots \\ (F_{11})_n & (F_{12})_n \end{bmatrix}_{(m^2 \times 2)};$$

$$F_2 = \begin{bmatrix} (F_{21})_1 & (F_{12})_1 & (F_{23})_1 \\ \dots & \dots & \dots \\ (F_{21})_j & (F_{12})_j & (F_{23})_j \\ \dots & \dots & \dots \\ (F_{21})_n & (F_{12})_n & (F_{23})_n \end{bmatrix}_{(m^2\times 3)};$$

$$D = \left\lfloor (D_1)_1^{\mathrm{T}} \dots (D_1)_j^{\mathrm{T}} \dots (D_1)_n^{\mathrm{T}}\ (D_2)_1^{\mathrm{T}} \dots (D_2)_j^{\mathrm{T}} \dots (D_2)_n^{\mathrm{T}} \right\rfloor^{\mathrm{T}}_{(2m^2\times 1)};$$

$$\theta = [q_1\ \ q_2\ \ q_3\ \ q_4\ \ q_5]^{\mathrm{T}}_{(5\times 1)};$$

O_1, O_2 – ($m^2 \times 2$) and ($m^2 \times 3$) matrixes, respectively, the elements of which are all zero;

$(D_1)_j$, $(D_2)_j$, $(F_{11})_j$, etc. – indicate the j^{th} column of the corresponding matrix.

Equation (18) can be solved for θ using least – squares technique.[23] The solution directly gives the estimation $\hat{k}_{\mathrm{S}}$ and remaining estimations are determined from the relations:

$$\hat{k}_{\mathrm{d}} = q_5/\hat{k}_{\mathrm{s}};\ \hat{\mu}_{\max} = \hat{k}_{\mathrm{d}} - q_4;\ \hat{Y}_{\mathrm{sx}} = q_2/\hat{\mu}_{\max}. \quad (19)$$

Experimental results

As it was mentioned above, the experiments are carried out only in one of the bioreservoirs and the experimental data have been received at every two hours. The duration of the process is 24 h. The length of bioreservoir is 162 m, while cross-section of the bioreservoir is 30 m^2. Six control points have been chosen in the bioreservoir, but complete data for variation of biomass and substrate are available only for three control points[16] – at the 4.6-th meter, 105-th meter and 162-th meter. The mass concentration of active sludge (biomass) has been measured[16] using sensor, based on the photometric analysis, with the error ±3 %. The mass concentration of organic contamination (substrate) has been calculated[16] based on the standard methods.

Using the identification algorithm, presented above, m – file in Matlab is created. Estimates of the parameters are as shown in Table 1.

Table 1 – *Estimated values*

$\hat{\mu}_{\max}$/h^{-1}	$\hat{k}_{\mathrm{d}}$/h^{-1}	$\hat{k}_{\mathrm{s}}$/g l^{-1}	$\hat{Y}_{\mathrm{SX}}$/g g^{-1}
0.1886	0.0114	0.025	0.1071

Figs. 2 and 3 present two points of view for the space distribution of substrate, when dispersion phenomena in the system are assumed to be negligible, and in the dependence of time t/h and bioreservoir length x/dm. Both, experimental data trajectories and the simulated ones from the model, described in analytical way by equation (6), are presented. Fig. 4, demonstrate as an example the substrate space distribution in the one of considered level, because the results in another three levels are similar. In the same way Figs. 5, 6 and 7 present the biomass space distribution, both, experimental data

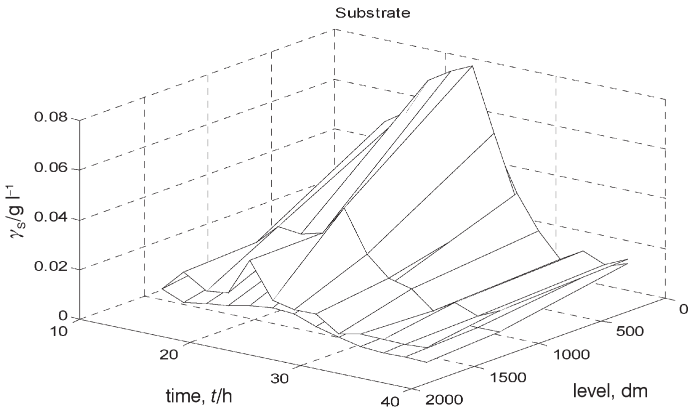


Fig. 2 – *Mass concentrations of organic contamination (substrate) γ_S/g l^{-1} – one view point*

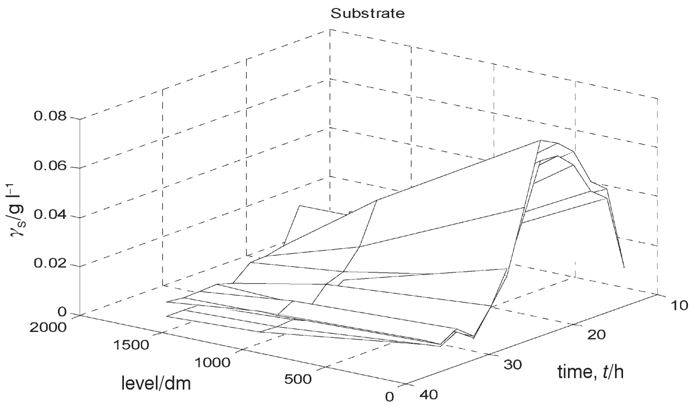


Fig. 3 – *Mass concentrations of organic contamination (substrate) γ_S/g l^{-1} – another view point*

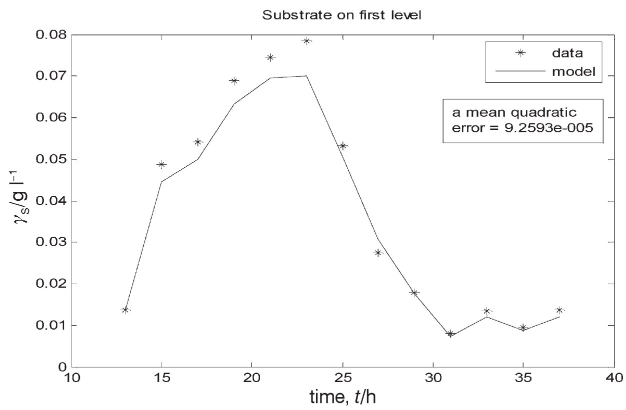


Fig. 4 – *Mass concentrations of organic contamination (substrate) γ_S/g l^{-1} – example for one level*

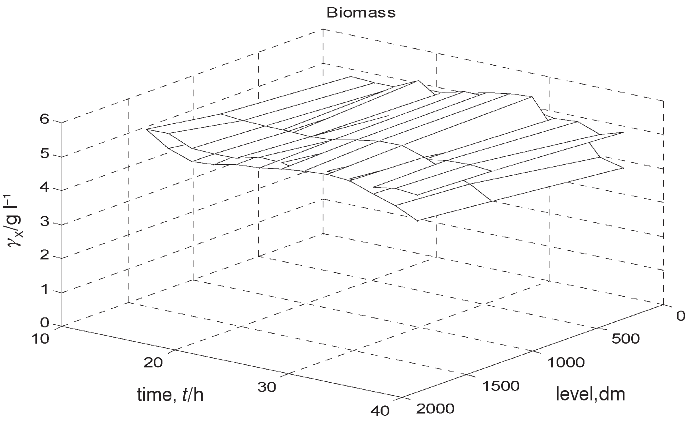


Fig. 5 – *Mass concentrations of active sludge (biomass) γ_X/g l^{-1} – one view point*

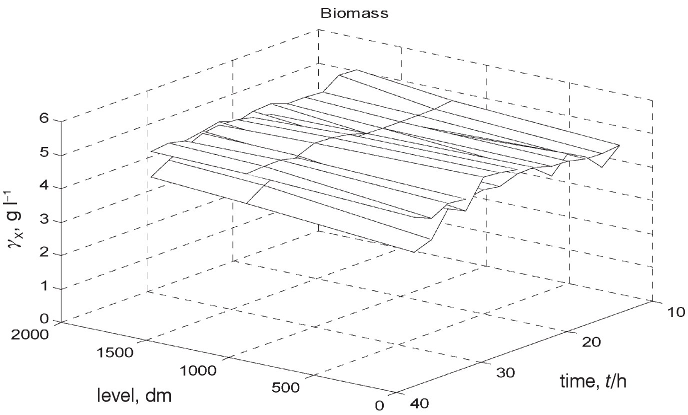


Fig. 6 – *Mass concentrations of active sludge (biomass) γ_X/g l^{-1} – one view point*

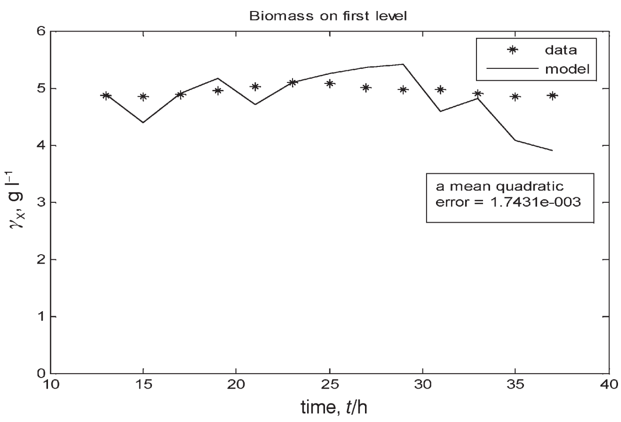


Fig. 7 – *Mass concentrations of active sludge (biomass) γ_X/g l^{-1} – example for one level*

trajectories and the simulated ones from the model, described in analytical way by equation (7).

Presented in figs. 2-4 and 5-7 results from the parameter identification of the mathematical model of the wastewater treatment process, described as object with distributed parameters, show high degree of adequateness of the obtained model and the efficiency of the presented in this paper of Haar orthogonal functions for parameter identification. As a proof for the adequateness of the obtained model, the values of Fisher criterion are presented[24] (Table 2).

Table 2 – *Fisher criterion*

Substrate	Fisher criterion	1.1500
	Table value at confidence probability 0.95 (α = 0.05)	1.83
	Table value at confidence probability 0.99 (α = 0.01)	2.34
Biomass	Fisher criterion	2.1991
	Table value at confidence probability 0.95 (α = 0.05)	2.27
	Table value at confidence probability 0.99 (α = 0.01)	3.28

Conclusions

The aim of this paper was the modelling and parameter identification of the wastewater treatment process, carried out in the operative system "bioreservoir-sedimentor" and described as object with distributed parameters. There are no familiar or accessible papers, in which this process has been considered as object with distributed parameters, with the exception of authors' papers.

A mathematical model, describing the dynamic and space variation of biochemical variables substrate S and biomass X in the wastewater treatment process in SBS, is obtained here on the basis of the material balance equations. Experimental data available from the operative wastewater treatment station gives a possibility for the identification, both, structural and parameter. As a result from implemented structural identification, the following conclusion could be reached, that the process dynamic description with the Contois' kinetics gives better results than the Monod' kinetics. Shifted two – dimensional Haar orthogonal functions have been used for parameter identification of developed model. Their implementation reduces the problem of parameter identification of the processes in SBS to a computationally convenient form. The algorithm is simple in form and with high precision. Presented results show, that the obtained mathematical model describes with a high degree of adequateness the wastewater treatment process, carried out in system "bioreservoir-sedimentor", and considered as object with distributed parameters in the case of negligible dispersion phenomena.

List of symbols

t – time coordinate
r, z, x – space coordinates

γ – continuous and differentiable function, describing mass concentration, g l^{-1}

$\boldsymbol{u}_r$, $\boldsymbol{u}_z$, $\boldsymbol{u}_x$ – components of the rate vector

$\gamma_S(x,t)$ – continuous and differentiable function, describing substrate mass concentrations, g l^{-1}

$\gamma_X(x,t)$ – continuous and differentiable function, describing biomass concentrations, g l^{-1};

Q – flow rate of wastewater, l h^{-1}

Q_a – flow rate of inlet air, l h^{-1}

S – bioreservoir cross- section, m^2

V_a – useful volume of bioreservoir, l

μ(S,X) – specific growth rate, h^{-1}

μ_{max} – maximum value of the specific growth rate, h^{-1}

k – yield factor, g g^{-1}

k_S – Michaelis' (saturation) constant, g l^{-1}

k_d – death coefficient of biomass h^{-1}

Y_{SX} – yield coefficient, g g^{-1}

D_{eff} – turbulent dispersion coefficient, m^2 h^{-1}

T – observed interval

$\boldsymbol{H}^*_M$ – vector with shifted Haar wavelets

Γ – reaction of system, g l^{-1} h^{-1}

Dimensionless symbols

r – quotient of capacities of recirculating active sludge and inlet wastewater;

ρ* – weight function;

k – integer positive number;

$\boldsymbol{C}^*_M$ – shifted two – dimensional Haar wavelets coefficients matrix;

c^*_{ij} – expansion coefficients.

Superscripts and subscripts

o, a, e – denote inlet, outlet and recirculation concentrations;

m – number of Haar wavelets;

i, j – integer positive numbers.

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