Optimal Design of Chemostats Connected in Series with Microbial Wall Growth

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The optimum design of chemostats, or continuous stirred tank reactors (CSTRs) connected in series with microbial wall growth is described. The optimum design of chemostats in series is based on the minimum overall reactor volume required to achieve a certain degree of substrate conversion. Topiwala –Hamer model for wall attachment is assumed and cell growth is described by Monod equation. Realistic values of cell concentration at the reactor wall are considered. The effect of kinetic and operating parameters namely, Monod constant, inlet substrate mass concentration, cell concentration at the reactor wall, substrate conversion and number of chemostats on the optimum design were determined. The effect of wall attachment on the total optimum dimensionless residence time is more pronounced at low inlet substrate concentration. The performance of the optimum design of a series of chemostats was compared with plug flow reactor and with the more convenient design of equal size chemostats required to achieve the same inlet and outlet conditions. The degree of benefit from using chemostats in series rather than one chemostat depends on the kinetic and operating parameters. The degree reduction (%) in the total volume using the optimum design compared to equal-size chemostats, can be up to 70 % depending on the kinetic and operating parameters. Up to five chemostats connected in series were described in this study.

Keywords:
Wall growth, Monod kinetics, chemostats, CSTRs in series, PFR, optimization.

Introduction

In recent years, a number of investigators have studied the optimum design of continuous stirred tank reactors in series for the performance of bioreactors of industrial interest.[1–7] Different kinetic equations have been used in the literature for the optimization of bioreactors in series for two cases: enzyme and autocatalytic system. Most researchers defined the optimum design as the minimum overall volume "holding time" of reactors required for a certain degree of substrate conversion and number of reactors. Chemostats in series are commonly used in biological treatment of industrial wastewater such as activated sludge basins, which are cascade connected.[8,9] This arrangement of chemostats offers number of advantages for degradations of toxic chemicals such as phenolic compounds.

Microbial growth on the walls of a bioreactor is important, especially, if the microbial concentration is not high, and in small bioreactors, which have large surface to volume ratio. In bioreactors with wall growth, the substrate is consumed by microorganisms, both, in the bulk liquid and attached to the reactor walls. Wall growth leads to higher concentrations of biomass and lower concentrations of substrate compared to that predicted by mathematical equations that describe chemostats. It is known that bacteria grow on the surface of reactor either in the form or monolayer or multi-layer.[10] The wall growth can be considered as a second, non-sterile feed, that prevents washout at high dilution rates. The attached microorganisms do not washout of the chemostat. The extent of wall growth depends on the degree of agitation in the bioreactor. The extent of wall growth is measured by the amount of biomass attached to the reactor surface per unit volume, X_w. A realistic value of X_w is between 0.02 and 0.05 g l^{-1}.[10] The bacterial attachment showed to have significant effects on the dynamics of the microbial predator-prey system[11]. Silicon coating of the inner surface of bioreactor showed to reduce the bacterial attachment by more than two folds.[11] Bacterial wall attachment showed to have considerable effect on stability behavior of bioreactors.[12] *Topiwala* and *Hamer*[10] analyzed the effect of wall growth on reactor performance for a single chemostat. *Pilyugin*, and *Waltman*[13] developed a model for chemostat with wall growth.

Little or no information is available in the literature about the effect of wall growth on the optimum design of chemostats in series.[3] The objectives of this work are to study the effect of wall growth, kinetics and operating parameters on the optimum design of chemostats connected in series. Microbial growth is assumed to follow Monod ki-

netics and the cell concentration at the reactor wall is assumed to be constant and not a function of the dilution rate in each vessel along the series. The effect of substrate feed concentration to the first reactor, the substrate conversion and Monod constant on the optimum design of reactors in series, were investigated. The optimum intermediate substrate concentration, the individual reactors holding time and the total holding time were determined and compared with the more convenient design of equal volume chemostats connected in series and PFR required for the same degree of substrate conversion. Up to five chemostats connected in series were used in this optimization study.

Theory

Optimization

The optimization procedure used here is similar to that used by *Luyben* and *Tramper*[1]. It is based on substrate balance over the cascade of reactors at steady state, followed by minimization of the overall reactors volume, required to achieve a certain degree of substrate conversion. Consider N chemostats connected in series with cell growth both in suspension and attached to the reactor wall. Derivations of the equations representing the optimum design of N chemostats in series are based on substrate balance on the i^{th} stage taking the following assumptions into considerations.

i) Cell growth is described by Monod kinetics. Kinetic parameters are assumed to be identical for all chemostats in series.
ii) Each stage is well mixed.
iii) Cell concentration on the reactor wall is constant in all stages.
iv) No cell death or cell maintenance.
v) Steady state is assumed.
vi) The specific growth rate and yield coefficient for cells on the reactor wall are assumed to be identical to that in suspension.
vii) There is no mass transfer limitations at the reactor wall.

Substrate balance over the i^{th} reactor at steady state gives:

$$Q(\gamma_{S_{i-1}}-\gamma_{S_i}) = \mu_i \gamma_{X_i} V_i / Y_x \qquad i = 1, 2, \ldots N \quad (1)$$

Where

Q Liquid volume flow rate, l h^{-1}
γ_S Substrate concentration, g l^{-1}
μ Specific growth rate, h^{-1}
γ_X cell concentration, g l^{-1}
V Reactor volume, l
Y_x Cell yields, $g_{cells}\ g^{-1}_{substrate}$.

Cell concentrations in each chemostate consist of two parts; cells in suspension and on the reactor wall, γ_{X_w}. Cell concentration in the i^{th} reactor can be related to the substrate concentration by the cell yield coefficient, $Y_{x,}$(taken as 0.5 in this work).

$$\gamma_{X_i} = \gamma_{X_0} + Y_x (\gamma_{S_0} - \gamma_{S_i}) \qquad i = 1, 2, \ldots N \quad (2)$$

Where:

γ_{X_0} is the cell mass concentration in the feed to the first reactor, g l^{-1}.

Using Monod kinetics, the mean residence time in the reactor $i(\tau_i)$ is given by:

$$\tau_i = Y_X \frac{\gamma_{S_{i-1}} - \gamma_{S_i}}{\frac{\mu_m \gamma_{S_i}}{K_S + \gamma_{S_i}} [\gamma_{X_i} + \gamma_{X_w}]} \qquad i = 1, 2, \ldots N \quad (3)$$

Substituting equation (2) into (3) assuming $\gamma_{X_0} = 0$ (sterile feed) and using dimensionless variables. The dimensionless residence time θ_i is given by:

$$\theta_i = Y_X \frac{(\alpha_{i-1} - \alpha_i)(K + \alpha_i)}{\alpha_i (A - a_i)} \qquad i = 1, 2, \ldots N \quad (4)$$

where

$$\alpha_i = \frac{\gamma_{S_i}}{\gamma_{S_0}}, \quad K = \frac{K_s}{\gamma_{S_0}}, \quad \theta_i = \mu_m \tau_i, \quad A = \frac{\gamma_{X_w}}{Y_X \gamma_{S_0}} + 1$$

where

θ : dimensionless residence time, $\tau \mu_m$
α : dimensionless substrate concentration, γ_S/γ_{S_0}
K : dimensionless Monod constant, K_s/γ_{S_0}

When wall growth is negligible in the reactor ($\gamma_{X_w} = 0$), A is equal to 1. To find conditions for the optimum design which is based on minimum overall reactors residence time.

$$\frac{\partial}{\partial \alpha_i}\left[\sum_{j=1}^{N} \theta_j\right] = 0 \qquad i = 1, 2, \ldots N\text{–}1 \quad (5)$$

Only two terms of equation (5) have α_i (i.e., i^{th} and $(i+1)^{th}$) as given by:

$$\frac{\partial}{\partial \alpha_i}\left[\frac{(\alpha_{i-1} - \alpha_i)(K + \alpha_i)}{\alpha_i (A - \alpha_i)} + \frac{(\alpha_i - \alpha_{i+1})(K + \alpha_{i+1})}{\alpha_{i+1}(A - \alpha_{i+1})}\right] = 0$$
$$i = 1, 2, \ldots N\text{–}1 \quad (6)$$

This equation reduces to

$$\frac{(\alpha_{i-1} - A)(K + \alpha_i)}{\alpha_i (A - \alpha_i)^2} - \frac{K(\alpha_{i-1} - \alpha_i)}{\alpha_i^2 (A - \alpha_i)} + \frac{(\alpha_{i+1} + K)}{\alpha_{i+1}(A - \alpha_{i+1})} = 0$$
$$i = 1, 2, \ldots N\text{–}1 \quad (7)$$

Equation 7 represents N–1 equations with N–1 unknowns (i.e., α_1 to α_{N-1}). α_0 by definition should be equal to 1 and α_N is related to the substrate conversion, δ by the relation $\delta = 1 - \alpha_N$. Equation 7 can be simplified to give α_{i-1} as a function of α_i and α_{i+1}:

$$\alpha_{i-1} = \frac{(K+A)\alpha_i^2 - \dfrac{\alpha_i^2(A-\alpha_i)^2(K+\alpha_{i+1})}{\alpha_{i+1}(A-\alpha_{i+1})}}{\alpha_i^2 + 2K\alpha_i - KA}$$

$$i = 1, 2, \ldots N\text{–}1 \qquad (8)$$

Knowing α_N and α_0 the intermediate dimensionless substrate concentrations that correspond to the optimum design can be calculated using the following algorithm:

1. Guess a value of α_{N-1}. ($\alpha_{N-1} = \alpha_N + \Delta$) where Δ is an increment such as 10^{-5}.

2. Knowing α_N, α_{N-1} calculate α_{N-2} from equation 8.

3. Knowing α_{N-1}, α_{N-2} calculate α_{N-3} from equation 8 and so on until α_0 is reached.

4. The computed α_0 is compared with 1, if they are not equal within 0.0001 a new guess of α_{N-1} is needed (new α_{N-1} = old α_{N-1} + Δ).

5. The above steps are repeated until $\alpha_0 = 1$.

6. The values of α_i, that gives $\alpha_0 = 1$ are the intermediate dimensionless substrate concentration that corresponds to the optimum design of reactors.

7. Knowing the values of α_i, the dimensionless residence time for each reactor can be calculated from equation 4. Guessing the value of α_{N-1} and confirm it with α_0 (Moving backwards) avoids the solution of quadratic equation which is obtained by guessing α_1 and continuing iteration to the last reactor where we confirm this guess by comparing α_N with the desired conversion.

A FORTRAN program based on the above algorithm was used to determine the values of α_i which define the optimum design of reactors.

Plug stream reactor with wall growth

The residence time for PFR with wall growth is determined by integration of Monod kinetic equation.[14]

$$\tau_{PFR} = -Y_x \int_{\gamma_{S_0}}^{\gamma_S} \frac{d\gamma_S}{\mu(\gamma_X + \gamma_{X_w})} = -Y_x \int_{\gamma_{S_0}}^{\gamma_S} \frac{d\gamma_S}{\dfrac{\mu_m \gamma_S}{K_s + \gamma_S}(\gamma_X + \gamma_{X_w})} \qquad (9)$$

Substituting for γ_X in this equation with Y_x ($\gamma_{S_0} - \gamma_S$) and using dimensionless variables, the dimensionless residence time for plug stream reactor is determined:

$$\theta_{PFR} = \left(\frac{K+A}{A}\right) \ln \frac{A-\alpha_L}{A-1} - \frac{K}{A} \ln \alpha_L \qquad (10)$$

Where

α_L is the dimensionless substrate mass concentration at the PFR exit ($\alpha_L = 1 - \delta$).

It is clear from equation 10 that the dimensionless residence time for PFR is infinity, when there is no cell growth at the reactor wall (i.e., $A = 1$ or $\gamma_{X_w} = 0$).

The critical dimensionless substrate concentration, $\alpha_{critical}$

Hill and *Robinson*[4] used the concept of $\alpha_{critical}$ to determine if there is advantage of using multi-stage chemostats. If α_1 approaches α_2, $\alpha = \alpha_{critical}$ and it is given by:

$$\alpha_{critical} = \sqrt{K^2 + K + K(A-1)} - K = f(K, A) \qquad (11)$$

It is clear from equation 11 that the critical dimensionless substrate concentration depends on the dimensionless Monod constant and the constant A. The required substrate conversion determines weather one or multiple chemostats should be used to minimize the total reactors volume.

If $\alpha_N < \alpha_{critical}$ Multiple chemostats are preferred

If $\alpha_{critical} \leq \alpha_N$ One chemostat is preferred

Chemostats of equal size

The volume of chemostats of equal volume (the currently practiced design criteria) was obtained and compared with the optimum volume of chemostats required to achieve a certain degree of substrate conversion.

Applying equation (4) for the chemostats i and i+1 (i.e equating θ_i and θ_{i+1}), the intermediate substrate concentrations, α_i can be obtained as a function of $\alpha_{i+1}$and α_{i-1}, which satisfies the conditions of equal size chemostats. The relation is given by:

$$\alpha_{i-1} = (\alpha_i/\alpha_{i+1})\{K+\alpha_{i+1}/K+\alpha_i\}[A-\alpha_i/A-\alpha_{i+1}](\alpha_i-\alpha_{i+1})+\alpha_i$$

$$i = 1,.. N\text{–}1 \qquad (12)$$

A FORTRAN computer program was used to calculate the intermediate substrate concentrations α_i for equal size chemostats. The total residence time can be calculated and is given by:

$$\theta_{tot,eq} = N\ \theta_{eq} \qquad (13)$$

The total residence time in the case of equal volume chemostats and non-equal volme (optimum or minimum volume) chemostats connected in series were compared. The degree reduction in total volume using minimum volume design as compared to equal volume design, was calculated

Degree reduction in total volume

$$(\%) = (\theta_{tot,eq} - \theta_{tot,opt})/\theta_{tot,eq} \times 100 \qquad (14)$$

Results and discussions

From the design equations above, it is clear that the optimum configuration for N chemostats in series depend on the following five quantities: The Monod constant, K_s; the cell concentration at the reactor wall, γ_{X_w}; the inlet substrate mass concentration, γ_{S_0}; the dimensionless exit substrate concentration, α_N and the number of chemostats connected in series, N. Table 1 shows the range of quantities used in this study. Mathematically, the effect of constant cell concentration at the chemostat wall, γ_{X_w}, on the performance of chemostat is equivalent to chemostat with non-sterile feed or with cells immobilized in a chemostat at constant concentration or chemostat with cell recycle.[3] For N chemostats in series, using typical kinetic and operating variables (γ_{X_w} = 0.03, γ_{S_0} = 1.0, K_s = 0.02, α_N = 0.01), tables 2 and 3 showed the calculated optimum intermediate substrate concentration and dimensionless residence time respectively.

Table 1 – *Range of quantities used in this work*

N (Number of chemostats in series)	1 to 5 in addition to PFR
γ_{S_0} (Inlet substrate concentration)	0.001 to 5 g l^{-1}
γ_{X_w} (Cell concentration at the reactor wall)	0 to 0.1 g l^{-1}
K_s (Monod constant)	0.001 to 0.5 g l^{-1}
α_N (Dimensionless exit substrate mass concentration)	0.001 to 0.1 (Conversion of 90 to 99.9 %)

Fig.1A (K_s = 0.02, γ_{S_0} = 1.0, α_N = 0.01) shows the effect of cell concentration at the reactor wall, γ_{X_w} on the total optimum residence time of N chemostats connected in series. Also the effect of γ_{X_w} on the residence time of PFR is shown on the same figure for comparison. As shown in Fig.1A, increasing γ_{X_w} reduced the residence time very little in case of N = 1 to 5, however, in the case of PFR the residence time strongly depends on γ_{X_w}. The residence time is infinity at γ_{X_w} = 0 (No wall growth) and decreased with increasing γ_{X_w}. At γ_{X_w} = 0.0375 g l^{-1} the PFR behaves like one chemostat. At the range of γ_{X_w} studied (0 to 0.1) which is typical of real chemostat systems,[10] two chemostats are preferred over PFR and even one chemostat is better than PFR at γ_{X_w} < 0.0375 g l^{-1}. In other words PFR is not recommended at very low cell concentration at the reactor wall. *Grieves* et al.[15] showed that PFR is better than a single CSTR except at very small values of constant A (i.e low γ_{X_w}) which is in agreement with the results in this work. It is also clear from Fig.1A that the largest reduction in residence time is when the number of chemostat increased from one to two. Very small reduction in residence time is obtained by increasing N from 2 to 3. Further increases of N is not economically feasible which suggest that N = 2 or 3 is the optimum number of chemostats using Monod kinetics. Similar results were reported by others.[3,4]

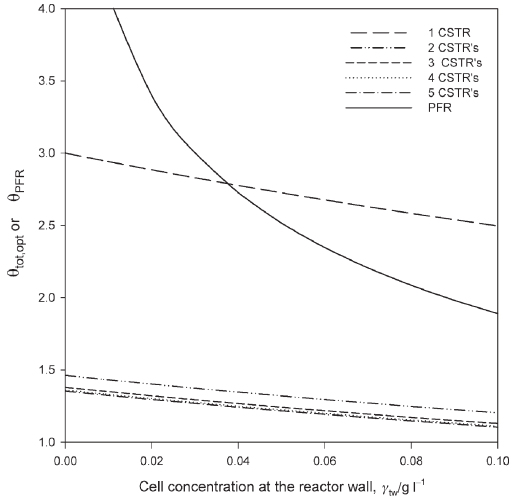
Table 2 – *The optimum intermediate dimensionless substrate concentration in cascade of N chemostats in series (γ_{X_w} = 0.03, γ_{S_0} = 1.0, K_s = 0.02, α_N = 0.01)*

N \ i	1	2	3	4	5
1	0.01				
2	0.0822	0.01			
3	0.1123	0.033	0.01		
4	0.1199	0.0506	0.0224	0.01	
5	0.1226	0.0623	0.0335	0.0183	0.01

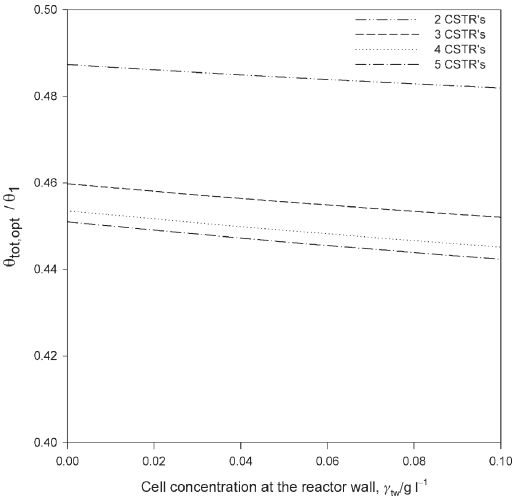
Table 3 – *The optimum dimensionless residence time for N chemostats in series. (γ_{X_w} = 0.03, γ_{S_0} = 1.0, K_s = 0.02, α_N = 0.01). θ_{PFR} = 3.0031*

N \ θ_i	1	2	3	4	5	$\theta_{tot,opt}$
1	2.8286					2.8286
2	1.1671	0.2063				1.3733
3	1.1034	0.1240	0.0658			1.2932
4	1.0924	0.0958	0.0514	0.0354		1.2750
5	1.0886	0.0799	0.0448	0.0307	0.0237	1.2676

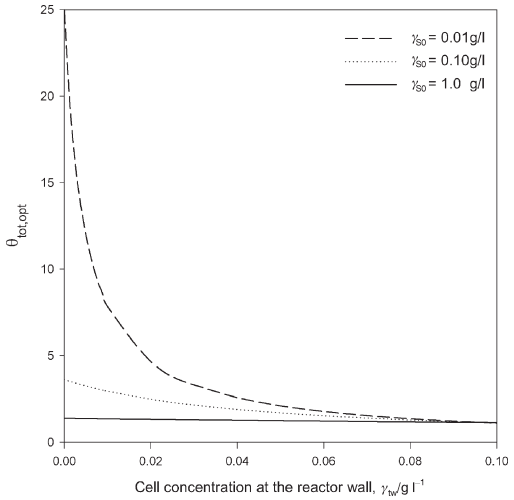
Fig.1B shows the effect of γ_{X_w} on the normalized dimensionless residence time which is the total dimensionless residence time of N chemostats divided by the dimensionless residence time for one chemostat required to achieve the same degree of



F i g . 1 A – *Effects of wall growth on the total optimum dimensionless residence time (K_s = 0.02, γ_{S_0} = 1.0, α_N = 0.01)*



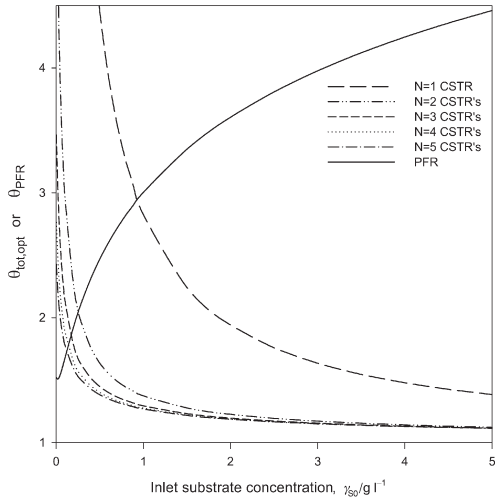
F i g . 1 B – *Normalized dimensionless residence time versus cell concentration at the reactor wall. (K_s = 0.02, γ_{S_0} = 1.0, α_N = 0.01)*



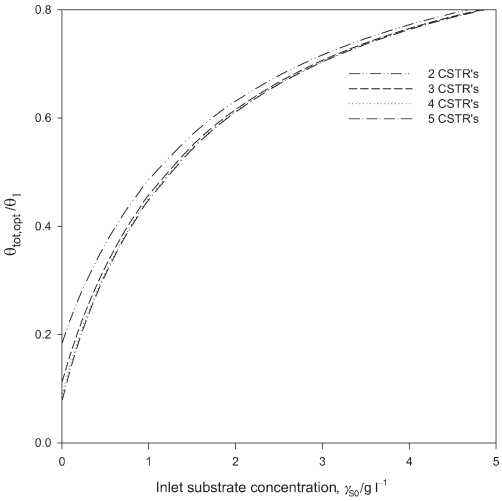
F i g . 1 C – *Effects of wall growth on the total optimum dimensionless residence time for three chemostats in series at different inlet substrate concentrations. (K_s = 0.02, α_N = 0.01)*

substrate conversion. As shown in Fig. 1B increasing γ_{X_W} decreases the residence time very little. Also two or three chemostats are recommended for cell growth described by Monod kinetics. Fig. 1C shows the effect of γ_{X_W} on the total optimum dimensionless residence time for three chemostats connected in series at different inlet substrate concentrations. It is clear from this figure that the effect of γ_{X_W} on the total residence time is important only at low inlet substrate concentration, γ_{S_0}. At high γ_{X_W}, γ_{S_0} has no effect on the total dimensionless residence time.

Fig. 2A shows the effect of inlet substrate concentration, γ_{S_0} on the total optimum dimensionless residence time for *N* chemostats connected in series (γ_{X_W} = 0.03, K_s = 0.02, α_N = 0.01). The performance of PFR is also shown for comparison. Increasing γ_{S_0}



F i g . 2 A – *Effects of inlet substrate concentration on the total optimum dimensionless residence time (γ_{X_w} = 0.03, K_s = 0.02, α_N = 0.01)*



F i g . 2 B – *Normalized dimensionless residence time versus inlet substrate concentration. (γ_{X_w} = 0.03, K_s = 0.02, α_N = 0.01)*

reduces the total residence time for N chemostats in series while for PFR increasing γ_{S_0} increases the residence time. At very low γ_{S_0}, the residence time for chemostats in series is very high ($\gamma_{S_0} \to 0$, $\theta_{tot,opt} \to \infty$). Fig. 2A shows that PFR behaves like a series of chemostats of N = 1,2,3,4 and 5 when the inlet substrate concentrations are 0.935, 0.258, 0.1715, 0.14 and 0.1215 g l^{-1} respectively. Fig. 2B shows the effect of γ_{S_0} on the normalized dimensionless residence time for N chemostats in series. This figure shows that chemostats in series are preferred at low γ_{S_0}, while at high γ_{S_0}, staging of chemostats is not preferred. Again two or three chemostats in series are the optimum choice.

Fig. 3A shows the effect of Monod constant, K_s on the total optimum dimensionless residence time (γ_{X_W} = 0.03, γ_{S_0} = 1, α_N = 0.01). Increasing K_s increases the total residence time. At very low K_s (zero order kinetics), the number of chemostats has no effect on the total residence time. Also increasing K_s increases the residence time for PFR.

From Fig. 3B it is clear that chemostats in series are preferred at high K_s and two or three chemostats in series are recommended.

Fig. 4A shows the effect of the dimensionless exit substrate concentration, α_N on the total optimum dimensionless residence time (for $\alpha_N < \alpha_{critical}$, γ_{X_W} = 0.03, γ_{S_0} = 1, K_s = 0.02). Increasing the substrate conversion increases the total residence time. The residence time increased sharply at high conversion only for single chemostat. At Fig. 4A conditions the PFR is not a good choice. It is clear also from Fig. 4B that chemostats in series are preferred at high substrate conversion. For $\alpha_N > 0.127 = \alpha_{critical}$, it is not feasible to use multiple chemostats in series. Single chemostat is recommended.

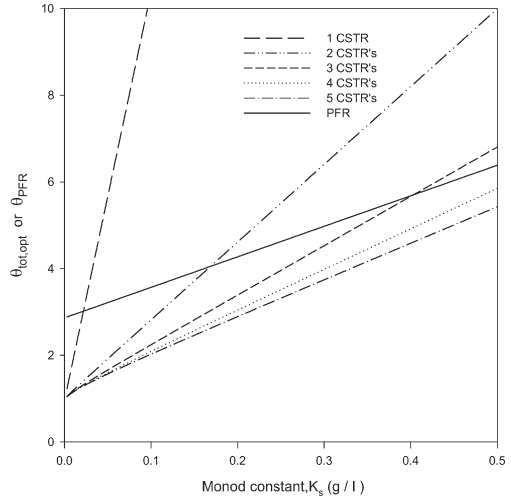


Fig. 3A – *Effects of Monod constant on the total optimum dimensionless residence time. (γ_{X_w} = 0.03, γ_{S_0} = 1.0, α_N = 0.01)*

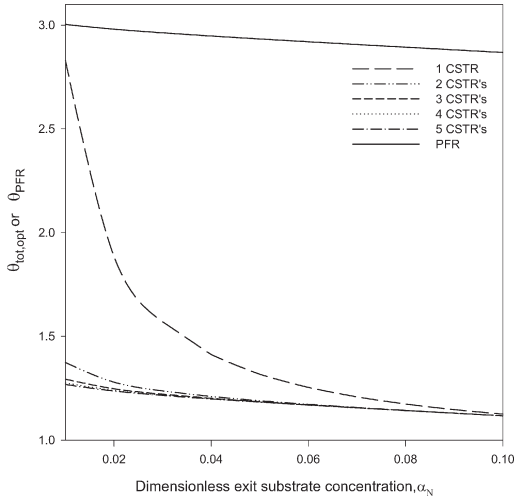


Fig. 4A – *Effects of substrate conversion on the total optimum dimensionless residence time. (γ_{X_w} = 0.03, K_s = 0.02, γ_{S_0} = 1.0)*

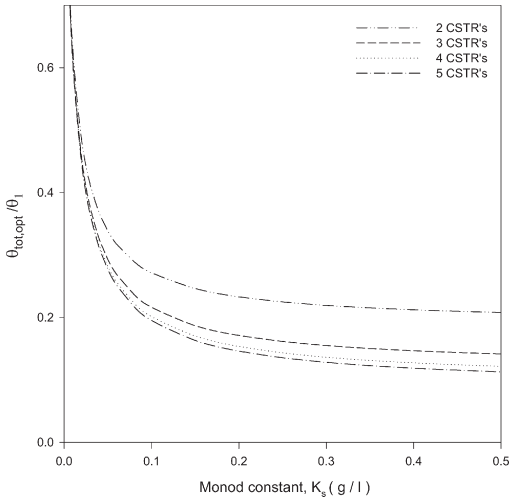


Fig. 3B – *Normalized dimensionless residence time versus Monod constant. (γ_{X_w} = 0.03, γ_{S_0} = 1.0, α_N = 0.01)*

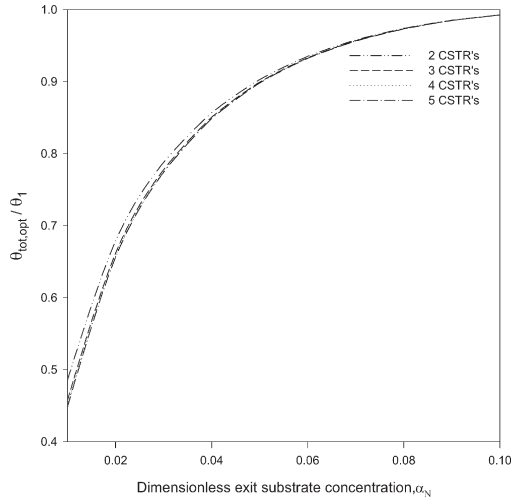


Fig. 4B – *Effects of substrate conversion on the normalized dimensionless residence time. (γ_{X_w} = 0.03, K_s = 0.02, γ_{S_0} = 1.0)*

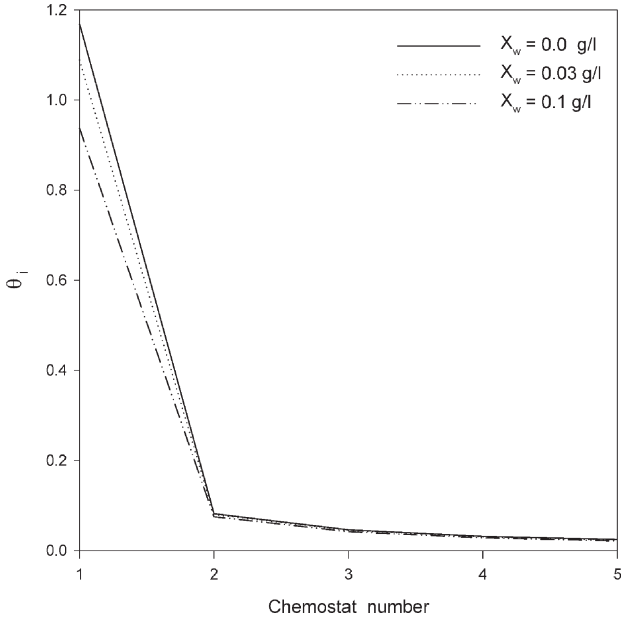


Fig. 5A – *Effects of wall growth on the individual dimensionless residence time for five chemostats connected in series. (K_s = 0.02, γ_{S_0} = 1.0, α_N = 0.01)*

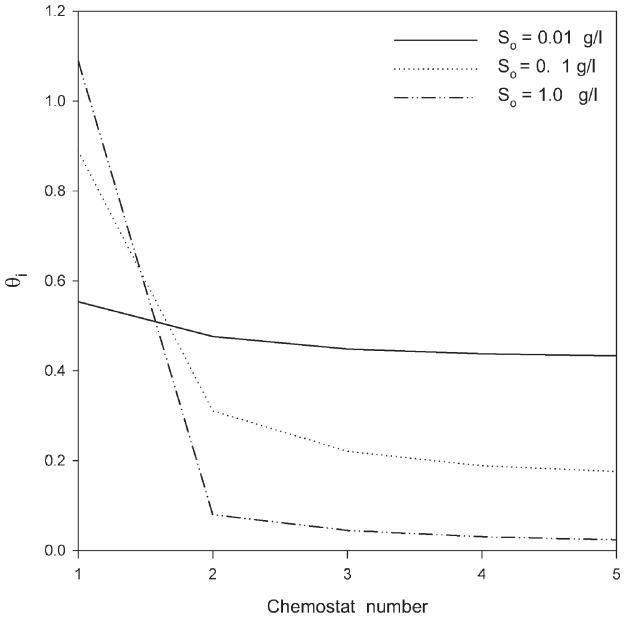


Fig. 5B – *Effects of inlet substrate concentration on the individual dimensionless residence time for five chemostats connected in series. (γ_{X_w} = 0.03, K_s = 0.02, α_N = 0.01)*

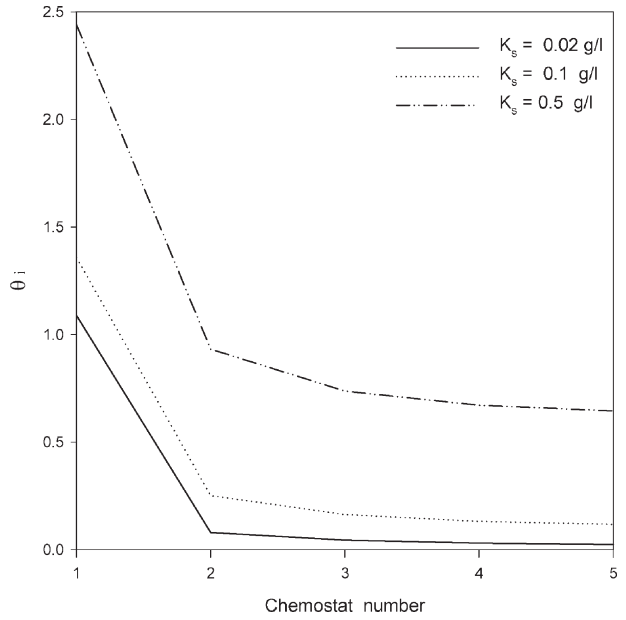


Fig. 5C – *Effects of Monod constant on the individual dimensionless residence time for five chemostats connected in series. (γ_{X_w} = 0.03, γ_{S_0} = 1.0, α_N = 0.01)*

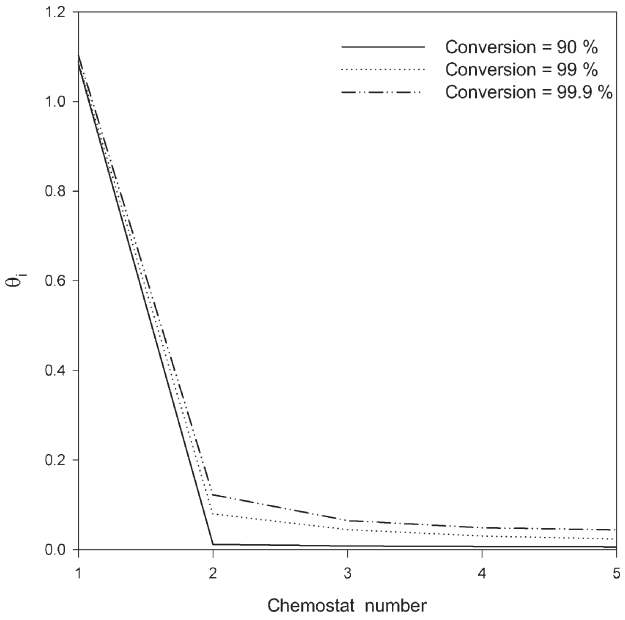


Fig. 5D – *Effects of substrate conversion on the individual dimensionless residence time for five chemostats connected in series. (γ_{X_w} = 0.03, K_s = 0.02, γ_{S_0} = 1.0)*

Figs. 5A-D show that for five chemostats connected in series at different kinetic and operating conditions, the volume of the first chemostat is the largest followed by smaller chemostats. Fig. 5B shows that for very small inlet substrate concentration, γ_{S_0}, all the chemostats have almost the same size. In Fig. 5D, it can be seen that for 90 % conversion (α_5 = 0.1), almost all conversion takes place in the first chemostat. Other chemostats volume are very small. In this case multiple chemostats are not recommended.

For chemostats of equal size connected in series, the total residence time was determined and compared with that of the minimum volume (optimum) design required to achieve the same degree of substrate conversion. Chemostats of equal size require larger total volume compared to the optimum non-equal number chemostats. The percentage reduction in total volume using the optimum design compared to equal volume design depends on γ_{X_w}, γ_{S_0}, K_s, α_N and N . The degree reduction in

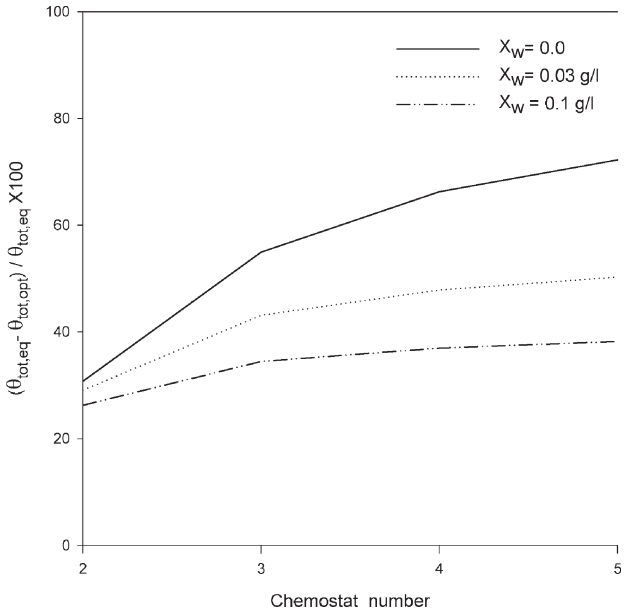


Fig. 6A – *Effects of wall growth on the degree reduction in total volume using optimum volume design as compared to equal volume design for 2,3,4 and 5 chemostats connected in series. (K_s = 0.02, γ_{S_0} = 1.0, α_N = 0.01)*

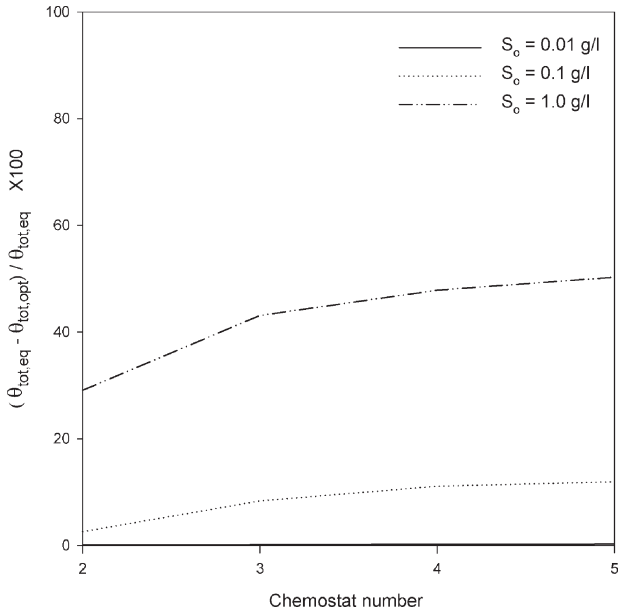


Fig. 6B – *Effects of inlet substrate concentration on the degree reduction in total volume using optimum volume design as compared to equal volume design for 2,3,4 and 5 chemostats connected in series. (γ_{X_w} = 0.03, K_s = 0.02, α_N = 0.01)*

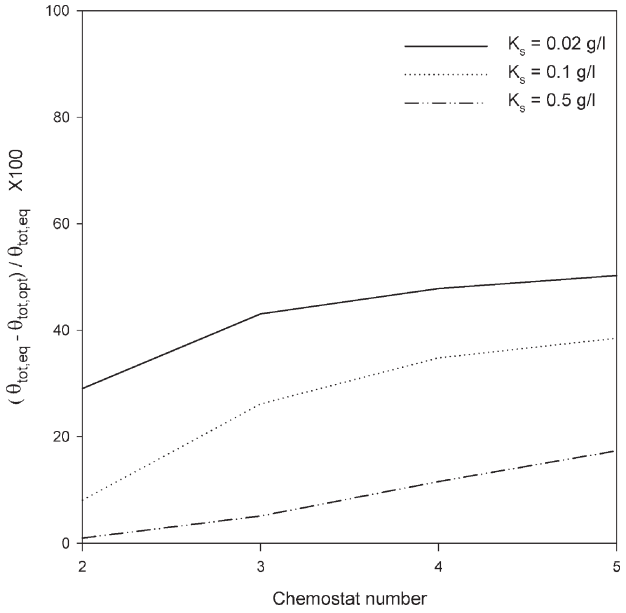


Fig. 6C – *Effects of Monod constant on the degree reduction in total volume using optimum volume design as compared to equal volume design for 2,3,4 and 5 chemostats connected in series. (γ_{X_w} = 0.03, γ_{S_0} = 1.0, α_N = 0.01)*

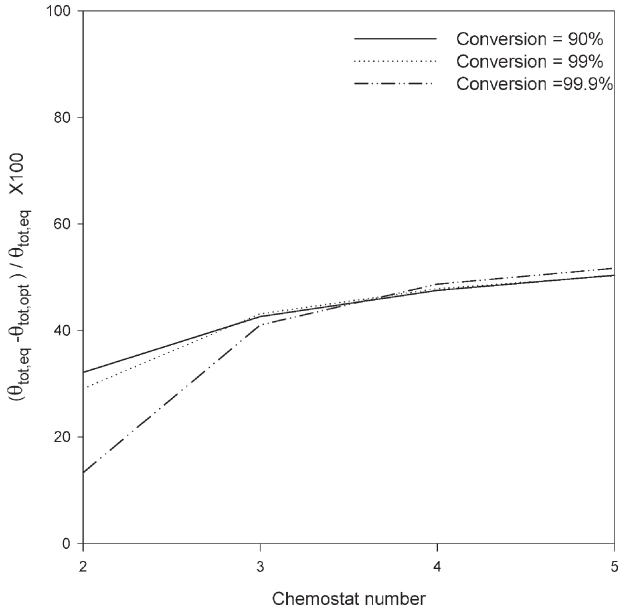


Fig. 6D – *Effects of substrate conversion on the degree reduction in total volume using optimum volume design as compared to equal volume design for 2,3,4 and 5 chemostats connected in series. (γ_{X_w} = 0.03, K_s = 0.02, γ_{S_0} = 1.0)*

total volume can be up to 70 %. As shown in Figs. 6A-D, the degree reduction in total volume increased with decreasing γ_{X_w}, K_s and increased with increasing γ_{S_0} and number of chemostats in series. The effect of conversion is appreciable only for two chemostats in series where high degree reduction is obtained at lower conversion. *Malcata*[16] reported a deviation of below 10 % between the two design criteria for CSTRs in series performing enzyme-catalyzed reactions described by Michaelis-Menten kinetics.

Conclusions

For *N* chemostats connected in series with wall growth and microbial growth described by Monod kinetics, the following conclusions can be drawn:

– The total optimum dimensionless residence time decreases with increasing the cell concentration at the reactor wall. The effect of wall attachment on the total dimensionless residence time is more pronounced at low inlet substrate mass concentrations. For PFR, the residence time decreases sharply with increasing the cell concentration at the reactor wall. At low cell concentrations at the reactor wall, the PFR is not a good choice compared to single or multiple chemostats.

– Increasing the number of chemostats in series reduces the total dimensionless residence time. However, there is no advantage of using more than two or three chemostats in series. Increasing the inlet substrate mass concentration or decreasing Monod constant reduces the total residence time for chemostats in series.

– A series of chemostats are more beneficial at low inlet substrate concentration, high Monod constant, and high substrate conversion.

– Using the optimum design criteria for a series of chemostats, the volume of the first chemostat is the largest followed by smaller ones. For very small inlet substrate concentration, the optimum design is chemostats of almost equal volume.

– The optimum volume design for chemostats in series is more efficient than the equal- volume design. The degree reduction in total volume, using the optimum design compared to the more convenient equal volume design, can be of up to 70 % at the conditions used in this work. The degree reduction in total volume increased with decreasing Monod constant and cell concentration at the wall or increasing the inlet substrate concentration and number of chemostats in series. The effect of conversion is appreciable only for two chemostats in series where high degree reduction is obtained at lower conversion.

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Nomenclature

A – constant defined as $\left(\frac{\gamma_{X_w}}{\gamma_{S_0} Y_x}+1\right)$

Q – liquid flow rate, l h^{-1}

K – dimensionless Monod constant.

K_s – Monod constant, g l^{-1}

N – number of chemostats in series

γ_S – substrate mass concentration, g l^{-1}

V – reactor volume, l

γ_X – cell mass concentration, g l^{-1}

Y_x – cell yield, g_{cells} $g^{-1}_{substrate}$

Greek symbols

α – dimensionless substrate concentration

τ – residence time, h

θ – dimensionless residence time,

μ – specific growth rate, h^{-1}

μ_m – maximum specific growth rate, h^{-1}

δ – degree of substrate conversion

Δ – increment (eg. 10^{-5})

Subscripts

eq – equal

i – refer to the i^{th} reactor

L – refer to the plug flow reactor exit

N – refer to the n^{th} reactor

0 – initial

PFR – plug flow reactor

tot – total

w – wall

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