The Higher Order Multilevel Fuzzy Logic Controller

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The higher order multilevel fuzzy controller was used to achieve an improvement of the performance of the auto controller tuning. A tuning method with higher order multilevel membership function is incorporated in a tuning formula based on the gain/phase margins. The higher order fuzzy multilevel controller was made to vary with respect to the plant's normalized dead time and system state error. The multilevel tuning actually has made the fuzzy logic controller able to improve adaptation of the control environment. The higher order multilevel controller was settled faster by the first order multilevel fuzzy logic controller in conjunction with less overshoot in set point control. As a case study the two coupled chemical stirred tanks were used. Numerical solutions showed validity of the proposed tuning methods. The obtained results were: illustrated feasibility of using the higher order multilevel fuzzy controller to represent tuning formula on the gain/phase margin for unstable zeros. The higher order multilevel controller was improved robustness and sharpness to determine gain/phase margin. This work is the first report in the literature showing the third order membership functions theory.

Key words:
Controller tuning, fuzzy logic, higher order fuzzy controller, nonlinear systems

Introduction

Recently, fuzzy logic controllers -FLC's have been successfully applied to a wide range of industrial processes as well as consumer products, and show certain advantages over the conventional PI and PID controllers.[1–5] On the other hand, although, fuzzy controllers have been extensively studied in control engineering, there are still rather few theoretical proofs that can explain why FLC's can achieve better performance.

Nowadays some studies have reported that, by replacing a conventional PI controller with a nonlinear fuzzy PI controller, better performance and local stability can be achieved. In these studies the weighting coefficients of the fuzzy logic controllers are calculated on the basis of a tuned conventional PI controller.

Thus, some guidelines for designing an FLC have been developed and theoretically proven. These rules and formula are helpful in eliminating the most time consuming trial-and-error procedures in the synthesis and design of fuzzy control systems. A control effort will be partly or fully saturated outside the universe of discourse.

In general, fuzzy logic control systems may have better system performance, but the complexity of the fuzzy rules base and the additional degree of freedom increase the difficulty of design. It is true that the weighting factors are functions of, both, parameter of the plant under control and the performance index of the closed loop system. In the design method based on gain and phase margins, it is also important to select a suitable equivalent gain/phase margin contour, so as to obtain appreciable performance. In previous studies[2–6] it was found that it is difficult to select such a contour, and the improper allocation of the equivalent contour will degrade the system's performance.

The multilevel fuzzy functions were applied[7,8] to trouble recognition and different kinds of faults classification. In previous papers[9,10] phase plane based analyses which was carried out showed that the multilevel modified tuning actually made the FLC able to adapt the control environment. The multilevel fuzzy logic controller – MFLC is more robust to handle nonlinear control problems.

This paper demonstrates higher order multilevel fuzzy control system. The higher order multilevel fuzzy logic controller – HMFLC shows the best performance in comparing with MFLC, simple FLC, and conventional PI controller with less overshoot in set point control. The HMFLC superior robustness was demonstrated. It could be used in the other domain.[11–15]

As a case study it is used the plant of two coupled reactors. Numerical simulations and experiments are presented to show validity of the proposed tuning methods. The improvement in system performance is confirmed through, both, simulation and experimental results. The obtained results have shown the advantages of the higher order multilevel fuzzy logic controller in robustness and parameter selectivity for nonlinear systems.

Desing and tuning of the multilevel fuzzy PI controller with membership function higher order

The block diagram of the fuzzy control system is shown in Figure 1. The plant can be approximated by a first-order plus dead time function and controlled by conventional PI controller. For the simple fuzzy logic controller, which has two inputs and one output, is developed the fuzzy PI controller with four fuzzy rules. The error and the change and error are defined as:

$$e(k) = r(k) - y(k) \quad (1)$$

$$\Delta e(k) = e(k) - e(k+1) \quad (2)$$

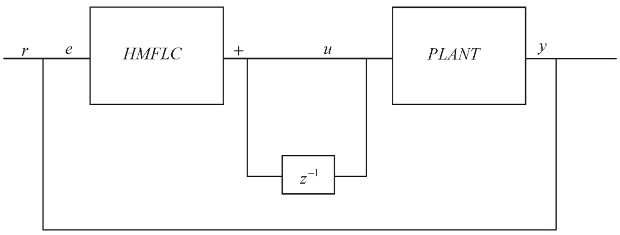

Fig. 1 – *Structure of the control loop*

the inputs of the fuzzy controller are the normalized error and the normalized change of error. The membership function $\mu(\boldsymbol{x})$[3,4] of the fuzzified inputs are of triangular shape (Appendix 1). The two fuzzy regions positive (P) and negative (N) for two input variables and the corresponding membership functions are defined as:

for positive fuzzy labels,

$$\mu(w_{x_i} x_i) = \begin{cases} 0 & w_x x_i < -1 \\ 0.5 + w_{x_i} x_i & -1 \le w_{x_i} x_i < 1 \\ 1 & w_{x_i} x_i \ge 1 \end{cases} \quad (3)$$

for negative fuzzy labels,

$$\mu(w_{x_i} x_i) = \begin{cases} 0 & w_x x_i < -1 \\ 0.5 - w_{x_i} x_i & -1 \le w_{x_i} x_i < 1 \\ 1 & w_{x_i} x_i \ge 1 \end{cases} \quad (4)$$

here is $x_i = (e, \Delta e)$. Consequently, the four simple fuzzy control rules, being used in FLC, are as follow:

IF (e is N) AND (Δe is N) THEN
(change in control is N)
IF (e is N) AND (Δe is P) THEN
(change in control is Z)
IF (e is P) AND (Δe is N) THEN (5)
(change in control is Z)
IF (e is P) AND (Δe is P) THEN
(change in control is P),

where the fuzzy labels of the control outputs are singeltons defined as $P = 1$, $Z = 0$ and $N = 1$.

Using the center of gravity defuzzification method (Appendix 1), the control output of the type of FLC can be obtained, when the normalized error or the normalized change of error are inside the universe of discourse (Appendix 2), as:

$$\Delta u = \frac{w_{\Delta u} w_{\Delta e}}{4 - 2\alpha} \left(\Delta e + \frac{\Delta t}{\frac{w_{\Delta e}}{w_e} \Delta t} e \right) = K_c^{(F)} \left(\Delta e + \frac{\Delta t}{T_i^{(F)}} e \right)$$

$$\text{with } \alpha = \max(w_e |e|, w_{\Delta e} |\Delta e|) \quad (6)$$

where $w_{\Delta u}$ is the scaling factor of the fuzzy control output. It can be concluded that this kind of basic fuzzy logic controller is a linear PI controller in structure, with a nonlinear proportional gain $K_c^{(F)}$ when inside the universe of discourse, and integral $T_i^{(F)}$. The control effort will be partly or fully saturated outside the universe of discourse. In the design method based on gain and phase margins, it is also important to select a suitable equivalent gain/phase margin contour, and the improper allocation of the equivalent contour degrade the system performance. To overcome this difficulty, multilevel fuzzy functions are included in the gain phase/margin tuning method, such that the equivalent gain/phase margin contour can be simply fixed. Fig.1 shows the structure of the fuzzy control system.

Remark 1. For every fuzzy subset A in the fuzzy set A a limited sequence fq with f-cut is determined, i.e.

$$0 < f_{q_1} < f_{q_2} < \ldots < f_{q_{p_{q-1}}} < 1 \quad (7)$$

The membership function of the second order (Appendix 3) can be structured by the composition by several membership function of the first order,

$$\mu^2(x_i) = \sum_{i=d}^{g} w_i \mu^1(x_i) \quad (8)$$

where $\sum_{i=d}^{g} w_i = 1$ and

$\mu^2(x_i) = \mu^2(x_d, \ldots, x_g), 1 \le d < g \le n$

In regard to the q-th group of parameter ($q = 1$) follows:

IF $f_1 \le \mu^2(x_i) \le 1$
THEN ***gain*** is **setting**,
IF $0 \le \mu^2(x_i) \le f_1$
THEN ***phase*** is setting. (9)

These two production rules, Eq.(9), improve design parameter selection.

Remark 2. The membership function of the third order can be structured by the composition of several membership functions of the second order,

$$\mu^3(x_i) = \sum_{i=1}^{b} w_i \mu^2(x_i) \qquad (10)$$

where $\sum_{i=1}^{b} w_i = 1$ and

$\mu^3(x_i) = \mu^3(x_1, \ldots, x_b), 1 \le l < b \le g$

In regard to the q-th group of parameter ($q = 1$) follows:

IF $f_1 \le \mu^3(x_i) \le 1$
THEN ***gain/phase margin*** is accurate. (11)

This production rule, Eq.(11), shows proper allocation of the equivalent gain /phase margin contour.

Description of the case study

The case study consists of two chemical stirred tanks as shown in Fig. 2. The basic task of the experiment is to control the reaction mixture level in the second stirred tank of the coupled system. The basic control strategy is to control the reaction mixture level in the second tank by varying the input flow to the first tank. The measurement for liquid level is read in, and control signal is written out. The control algorithm is realized by computer programming. Fig. 2 shows the process plant .

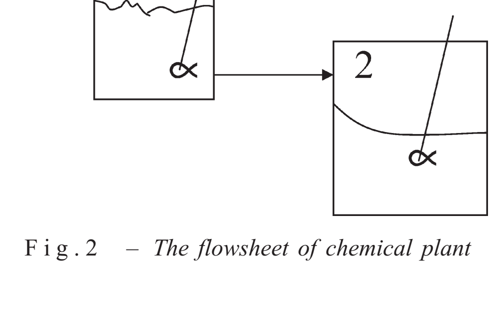

Fig. 2 – *The flowsheet of chemical plant*

The plant could be linearized, simplified and modeled by a first order plus dead time structure. The tuning of the fuzzy PI controller is based on an approximated model of the plant, it is similar to the method of tuning a conventional PI controller. It is assumed that the process under control can be modelled as first order plus dead time dynamics, and that all the three plant parameters (time constant, open-loop gain and dead time) can be obtained:

$$G(s) = \frac{K_c \exp(-Ls)}{1 + \tau s} \qquad (12)$$

For this kind of plant, by using the phase/gain margin tuning formula for a conventional PI controller and substituting the *equivalent* proportional gain and integral time the following equations (13) can be obtained:

$$w_e = \frac{0.2}{|r_0|}$$

$$w_{\Delta e} = \frac{1}{\left(2w_F - \frac{4w_F^2 L}{\pi} + \frac{1}{r}\right)\Delta t} w_e$$

$$w_{\Delta F} = \frac{w_F \tau}{A_m K_F w_{\Delta e}}(4 - 2\alpha_0) \qquad (13)$$

with

$$w_F = \frac{A_m \Phi_m + (\pi/2) A_m (A_m - 1)}{(A_m^2 - 1)L}$$

where r_0 is the set point changing range, A_m is specified gain margin, ϕ_m is the specified phase margin and w_p is the resulting phase crossover frequency.

In equation (13), a_0 is design parameter which can be selected from 0 to 1. Since:

$$a_0 = \max(w_e |e_0|, w_{\Delta e} |\Delta e|), \qquad (14)$$

It is easy to find out that this design parameter is related to certain points on the normalized phase plane and this can be interpreted as an equivalent gain/phase margin contour (Fig. 3).

According to the tuning formula, the weighting factors w_e, $w_{\Delta e}$, $w_{\Delta u}$ will be fixed and thus

$$\frac{w_{\Delta e} \Delta t}{w_e} = T_i^{(F)}$$

$$\frac{w_{\Delta u} w_{\Delta e}}{4 - 2\alpha} = \frac{4 - 2\alpha_0}{4 - 2\max(w_e|e|, w_{\Delta e}|\Delta e|)} \frac{1}{4 - 2\alpha_0} w_{\Delta u} w_{\Delta e}$$

$$= \gamma \frac{1}{4 - 2\alpha_0} w_{\Delta u} w_{\Delta e} \qquad (15)$$

$$= \gamma K_{c\alpha_0}^{(F)}$$

where $K^{(F)}_{c\alpha_0}$ is the gain of the fuzzy logic controller when the system is at its contour.[2,9,10] Clearly, the fuzzy controller-FLC has the property that its is fixed and is variable in terms of different e and Δe. The property of a_0 can be used to allocate the equivalent gain/phase margin contour so as to modify the closed loop system's performance. The multilevel fuzzy controller -MFLC[9,10] goes further in sensitivity varying gain or phase, alternatively according to equation (9) in terms of different e and Δe. The higher order multilevel fuzzy logic controller-HMFLC shows higher robustness and sharpness to allocate the equivalent gain/phase margin contour, according to equation (9) and (11) so as to modify the closed loop system performance. This improvement of conventional MFLC[9,10] with HMFLC is very important for the system with unstable zeros. Fig. 3 shows equivalent gain/phase margin contours for different a_0.

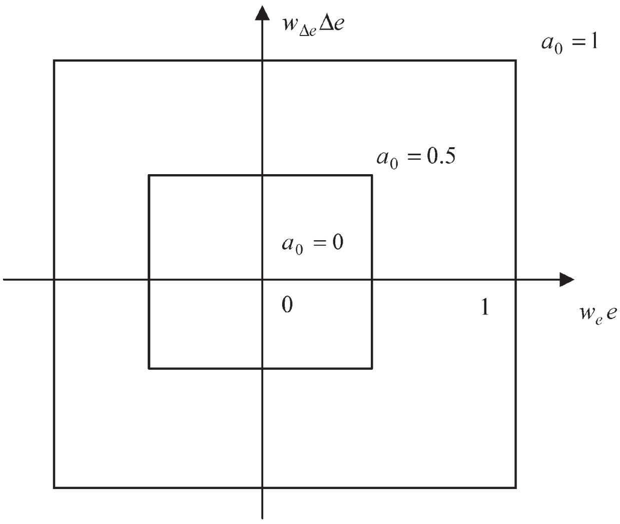

Fig. 3 – *Equivalent gain/phase margin contours for different a_0*

The main contribution of this paper is the tuning of a higher order multilevel fuzzy logic controller. The membership function third order defines common action of design parameters.

The resulting HMFLC appears to be a *varying parameter* PI controller which is a realization of the fuzzy control rules equations (5) and fuzzy parameters selection rules equations (7) to (11). Although qualitatively it is known that *ao* should be larger when the normalized dead time L/τ is large, the desired variation of *ao* may not be monotonic against L/τ.

The switching line of the fuzzy control action on normalized phase plane is:

$$w_e e = w_{\Delta u} \tag{16}$$

On the switching line the control action is zero. When the system state is across the switching line, the control action will change its sign.

Results and discussion

To verify effectiveness of the higher order multilevel fuzzy modification simulations experiments were carried out, in which conventional fuzzy logic and multilevel fuzzy logic with gain/phase margin tuning, were compared with the higher order multilevel fuzzy tuning formula. All the controllers were tuned using the same pair of gain and phase margin specifications of 3 and 45°, since this pair gives a good system response to both set-point change and load disturbance.

The first examined example is to control a plant with small normalized dead time. Controller parameters in the simulation to $G(s) = \exp(-3s)/(1+50s)$ for Δt = 1 are shown in Table 1. The simulation results are shown in Fig. 4. From this results, it is found that the conventional PI controller introduces large overshoot in the closed loop system, while the fuzzy controller makes the closed loop system a bit sluggish. With multilevel fuzzy tuning algorithm, the closed loop system has only a slight overshoot, and converges quickly to the set point. The values of the second order membership functions show which parameters can be fixed or varied. For example, if the value of membership function second order for gain margin is higher from the value of membership function second order for phase margin, then gain is varied and phase margin is fixed. The improvement multilevel fuzzy of settling time is around 40 % in comparison with conventional fuzzy PI. Also, better system respon-

Table 1 – *Controllers parameters in the simulation to G(s) = exp(–3 s)/(1+50 s)*

Controller	*Kc*	*Ti*	*we*	*w*Δ*e*	*w*Δ*u*
PI	12.42	9.05	–	–	–
FCL(a_0 = *0*)	–	–	0.200	18.1	2.21
MFLC	–	–	0.200	18.1	2.21
HMFLC					

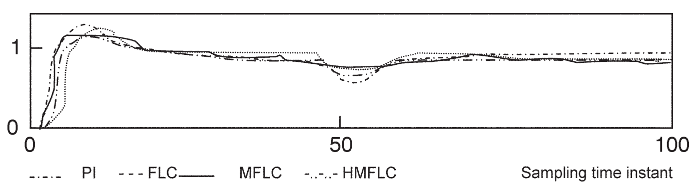

Fig.4 – *Closed loop responses (G(s) = exp(–3 s)/(1 + 50 s))*

se to load disturbance can be observed. The improvement higher order multilevel order tuning algorithm in comparison with multilevel PI is in proper allocation of the equivalent gain/phase margin contour. The membership function third order is determined gain/phase margin resolution. The closed loop system has shown higher robustness.

The second example is to control a plant with the same dead time and time constant as the first case, but with the different steady state gain of 10. The simulation results show that the closed-loop system responses are almost identical to those of the previous case, except for those to load disturbance. Controller parameters for this case in the simulation to $G(s) = 10\ \exp(-3\ s)\ /\ (1 + 50\ s)$ for $\Delta t = 1$ are given in Table 2.

Table 2 – *Controllers parameters in the simulation to G(s) = 10 exp(–3 s)/(1+50 s)*

Controller	*Kc*	*Ti*	*we*	*w*Δ*e*	*w*Δ*u*
PI	1.242	9.05	–	–	–
FLC(*ao=0)*	–	–	0.200	18.1	0.221
MFLC	–	–	0.200	18.1	0.221
HMFLC	–	–	0.200	18.1	0.221

The third example is to control a plant that has a large normalized dead time. Controller parameters in the simulation to $G(s) = \exp(-90\ s)\ /\ (1 + 50\ s)$ for $\Delta t = 1$ are given in Table 3. The simulation results are shown in Fig. 5. These results show that both conventional PI and fuzzy controlled systems have evident overshoots when set point changes. On the other hand, the mulitilevel fuzzy algorithm controls the system with only slight overshoot, and thus the system can be settled quickly at the set point. The improvement in settling time is 45 %. The higher order multilevel fuzzy systems improve allocation equivalent to phase/gain margin contour. The system responses to load disturbance are similar for the three controllers.

Table 3 – *Controllers parameters in the simulation to G(s) = exp(–90 s)/(1+50 s)*

Controller	*Kc*	*Ti*	*we*	*w*Δ*e*	*w*Δ*u*
PI	0.313	45.05	–	–	–
FLC (*ao=0)*	–	–	0.200	90.1	0.0139
MFLC	–	–	0.200	90.1	0.0139
HMFLC	–	–	0.200	90.1	0.0139

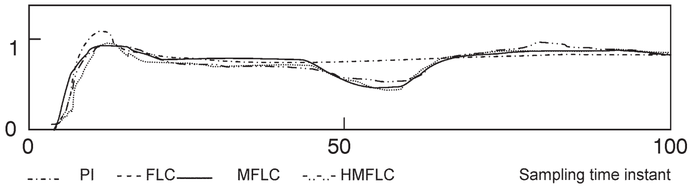

Fig. 5 – *Closed loop responses (G(s) = exp(–90 s)/(1 + 50 s)*

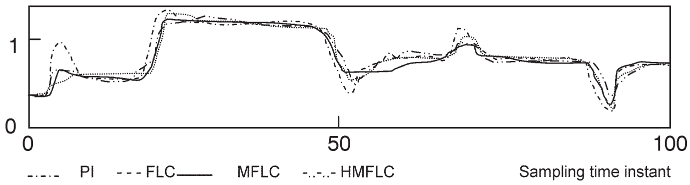

Fig. 6 – *The experimental closed-loop responses (G(s) = exp(–11.2 s)/(1 + 395 s)*

The experimental results for the chemical plant in Fig 2 are shown in Fig.6. In order to control the reaction mixture level in the second reactor it was varying the input flow speed in the first reactor.

The system is a nonlinear second order plant. For the design of controllers using gain and phase margin specifications, the plant can be linearized, simplified and modeled by first order and dead time. For a second order plus dead time plant, a fuzzy PID can be used. Thus a simplified plant model is $G(s) = 1.55\ \exp(-5\ s)\ /\ (1 + 395\ s)$.

The experiments are carried out during a period from 0 to 1200 s. There are set point changes at time instants of 30. Moreover, there are two load disturbances at time instant 880 and 1100, which is a 6 $cm^3\ s^{-1}$ flow introduced and removed to the first reactor to emulate the change in inflow. The specified gain and phase margin are 3 and 45°, respectively. The sampling interval is 1*s*.

Conclusions

The obtained results show that the performance of the higher order multilevel fuzzy controller is the best among those of three fuzzy kinds of controllers: The system reaches the set point faster with less overshoot, hence the settling time is the shortest, especially for unstable regions. Generally, fuzzy control is nonlinear in nature and is more robust to handle nonlinear control problem.

The underlying idea of the multilevel fuzzy logic controller is to associate the integral action and the proportional action of the fuzzy logic controller with the normalized dead time and state error. The multilevel fuzzy logic controller gives almost identical system response in all circumstances.

With higher order multilevel fuzzy logic closed – loop, performance for systems with either large or small normalized dead times are improved. The higher order multilevel fuzzy logic controller shows higher robustness and sharpness, and improves proper allocation of the suitable equivalent gain/phase margin contour. The improper allocation of the equivalent contours degrade the system performance. This paper is the first report in the literature showing the third order multilevel fuzzy controller. This controller can be applied in the other domain for those processes when biochemical reaction occurs.

Notation

A – fuzzy subset
A – fuzzy set
e – error
Δe – change of error
f – experienced factor (0.8)
L – dead time
L/τ – normalized dead time
N – negative fuzzy label
q – number of sequences
P – positive fuzzy label
r_0 – set point
t, T – time, integral time, respectively
K – proportional gain
u – fuzzy control output
x – input variable
Y – output variable
w – weighting factor

Subscript

(F) – fuzzy

Index

c – control
e – error
Δe – change of error
i – integral
q – number of sequences (fragments)
p – crossover frequency

Greek symbols

α_0 – design parameters
Am – specified gain margin
$\mu^1(..)$ – fuzzy membership function of the first order
$\mu^2(..)$ – fuzzy membership function of the second order
$\mu^3(..)$ – fuzzy membership function of the third order
Φ_m – specified phase margin
τ – time constant of the plant

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APPENDIX 1

Fuzzy set theory

Fuzzy sets are sets in which members are presented as ordered pairs that include information on degree of membership.[1,2] Let, introduce a fuzzy subset A of traditional set

$$U(u_1,\ u_2,\ u_k).$$

$$A = [u_i,\ \mu_A(u_i)],\ u_i = e\{A\} \quad \text{(A1.1)}$$

where $\mu_A(u_i)$ is degree of membership u_i in the subset A, and

$$\mu_A(u_i) = e\{0,1\}. \quad \text{(A1.2)}$$

If $\mu_A(u_i) = 0$ then u_i is not member of the subset A, and (A1.3)

If $\mu_A(u_i) = 1$ then u_i is member of the subset A, full membership. (A1.4)

A classical set of, say k elements, is a special case of a fuzzy set, where each of those k elements has 1 for the degree of the membership, and every other element in the classical set has a degree of membership 0, for each reason you don't bother to list it.

Fuzzy logic is combination of multivalued logic, probability theory, and artificial intelligence. It incorporates the imprecision inherent in many real world systems, including human reasoning, by allowing linguistic variable classifications such as big, slow, near zero or too fast. Unlike binary logic, fuzzy systems do not restrict a variable to be a member of a single set, but recognize that a given value may fit to varying degrees, into several. For example, a speed of 60 km h^{-1} may be moderately slow, fast or too fast depending on the other factors such as speed limit or road conditions.

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APPENDIX 2

Fuzzy set operations

Consider a union of two traditional sets and an element that belongs to only one of these sets. If these sets are treated as fuzzy sets this element has degree of membership equal 1 in one case and 0 in the other, since it belongs to one set and not the other. Let put this element in the union. It should be, to look at the two degree of membership namely, 0 and 1, and pick the higher of the two, namely 1. In the other words, the maximum values of its degrees of membership within the two fuzzy sets, forming a union.

For example,

$$x + y = \max\ (x,\ y) \quad \text{(A2.1)}$$

or

$$0 + 1 = \max\ (0,\ 1) = 1 \quad \text{(A3)}$$

$$1 + 1 = \max\ (1,\ 1) = 1$$

Analogously, the degree of membership of an element in the intersection of two fuzzy sets is the minimum or the smaller value of its degree of membership individually in the two sets forming the intersection. For example,

$$x\ y = \min\ (x,\ y) \quad \text{A(2.2)}$$

or

$$0 \times 1 = \min\ (0,\ 1) = 0$$

$$1 \times 0.8 = \min\ (1,\ 0.8) = 0.8.$$

In the fuzzy recording method a fuzzy membership value is appended to the need value, so this process can be refereed to as fuzzification. The main difference between the crisp forecast and the fuzzy forecast is that the former predicts not only the class value but also the values corresponding to the membership function. From the resulting qualitative variables continuous signal can then be regenerated and then subsequently be used as inputs to other quantitative or qualitative variables. This regeneration process is called defuzzification.

APPENDIX 3

Modelling approach by multilevel fuzzy functions

Fuzzy set theory, in effect, is a step toward a rapprochement between the precision of classical mathematics and the pervasive imprecision of the real world. Fuzziness of a phenomena steams from the lack of clearly defined boundaries.

Let set A and subset A_j'

$$A,\ A_j'(j = 1,\ 2,\ \ldots,\ m),\ A_j'\ e(A) \quad \text{(A3.1)}$$

be the output, global observation, set and subset, which contain various states to be diagnosed. Since output states in complex processes are often inconclusive, fuzzy set and fuzzy subsets, are applicable.

Assume that the observed field is a measurable output vector space consisting of n vectors:

$$X_i = (x_1,\ x_2,\ \ldots,\ x_n) \quad \text{(A3.2)}$$

where X_i is the i-th vector with which A can be ambiguously predicted, i.e., the subset A_j' can be determined according to their values of

$$X_{ij}(i = 1,\ 2,\ \ldots,\ n,\ j = 1,\ 2,\ \ldots,\ m).$$

Suppose that m fuzzy subsets are divided into q groups by various characteristics such as the kinds of parameters:

$$A = (A_{11}',\ \ldots,\ A_{1p}'),\ \ldots,\ (A_{k1}',\ \ldots,\ A_{kp}') \quad \text{(A3.3)}$$

where $\sum_{i=1}^{k} p_i = m.$

The reason why some subset in fuzzy set are combined into a group is, that there are some connections between them to be considered.

Any fuzzy subset A'_j of X_i is characterized by a membership function μ_{A_j} which associates with every member x_i, i.e., $\mu_{A_j}(x_i)$ representing the degree of membership of x_i to fuzzy subset A'_j.

A definition for the construction of a membership function is described as follows:

For every fuzzy subset $(A'_{q1},\ldots,A'_{qp})_q$ for $1 < q < k$ in the fuzzy set A given $A = (A_{l1},\ \ldots,\ A_{lp1}),\ \ldots,\ (A_{kl},\ \ldots,\ A_{kp})_k$ by a limited sequences f with f-cut is determined, i.e.:

$$0 < f_{q1} < f_{q2} < f_{q3} < \ldots < f_{q\ p-1} < 1 \quad \text{(A3.4)}$$

Corresponding membership function. The determination of the corresponding membership function must have the following relationship:

$$\text{IF } f_{qp} < \mu_{qp}(x_i) < 1 \text{ THEN } x_i \ e\{A'_{qp}\}_q$$
$$\text{IF } f_{q\,p-2} < \mu_{q\,p-1}(x_i) < f_{q\,p-1} \text{ THEN } x_i \ e\{A'_{q\,p-1}\}_q \quad \text{(A3.5)}$$
$$\text{IF } 0 < \mu_{ql}(x_i) < f_{q1} \text{ THEN } x_i \ e\{A'_{ql}\}$$

where $x_i \ e\{A'_j\}$ means that it is satisfied by the condition in which the states A' appears. Therefore, the membership functions in the sense of j.

Eq.(A3.5) are divided into several levels, such as μ_q having p_q levels. Since it is possible the $\mu_{A''_j}$ is a function of multivariable $\{X(i = 1, 2, \ldots, n)\}$, the membership function of the first order is denoted.

Analogously, the membership function of the first order can be structured into a membership function second order. The membership function of the second order

$$\mu^2_{A'_j}(x_d, \ldots, x_g), \ 1 < d < g < n$$

can be generated by the composition of several membership functions of the first order, i.e.:

$$\mu^2_{A'_j}(x_i) = \sum_{i=d}^{n} w_i \mu_{A'_j}(x_i) \quad \text{(A3.6)}$$

where w_i is weight factor whose value depends on the degree of the relationship between x_i and A'_j, and satisfied with

$$\sum_{i=d}^{n} w_i = 1 \quad \text{(A3.7)}$$

It is obvious that the membership functions of the second order have some similar characteristics with the membership function of the first order, i.e. corresponding to the q-th group of subsets

$$(A_{q\,l}, \ldots\ldots, A_{q\,p})_q, \ 1 < q < k \quad \text{(A3.8)}$$

there are relationships:

$$\text{IF } f_{q\,p} < \mu^2_{q\,p}(x_i) < 1 \text{ THEN } x_i \ e\{A'_{q\,p}\}_q$$
$$\text{IF } f_{q\,p-2} < \mu^2_{q\,p-1}(x_i) < f_{q\,p-1} \text{ THEN } x_i \ e\{A'_{q\,p-1}\}_q \quad \text{(A3.9)}$$
$$\text{IF } 0 < \mu^2_{q\,l}(x_i) < f_{q\,p} \text{ THEN } x_i \ e\{A'_{q\,l}\}.$$

These membership functions can be structured into a membership function of the third order. The membership function of the third order $\mu^3_{A'_j}(x_{A'}, \ldots, x_l)_b$, $d < l < b < g$ can be generated by the composition of several membership functions of the second order i.e.:

$$\mu^3_{A'_j}(x_i) = \sum_{i=p}^{q} w_i \mu^2_{A'_j}(x_i) \quad \text{(A3.10)}$$

where w_i is a weight factor whose value depends on the degree of the membership between x_i and A'_j, and satisfied by

$$\sum_{i=l}^{b} w_i = 1 \quad \text{(A3.11)}$$

The membership functions of the third order have some similar characteristics with the membership function of the second order, i.e. corresponding to q-th group of subsets $(A_{q1}, \ldots, A_{q\,p})_q$, $1 < q < k$ there are relationships given by

$$\text{IF } f_{q\,p} < \mu^3_{q\,p}(x_i) < 1 \text{ THEN } x_i \ e\{A'_{q\,p}\}_q$$
$$\text{IF } f_{q\,p-2} < \mu^3_{q\,p-1}(x_i) < f \text{ THEN } x_i \ e\{A'_{q\,p-1}\}_q \quad \text{(A3.12)}$$
$$\text{IF } 0 < \mu^3_{q\,l}(x_i) < f_{ql} \text{ THEN } x_i \ e\{A'_{q\,l}\}.$$