An Improved Autotune Identification Method

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A method is proposed to estimate accurately the model parameters of a first order plus time delay (FOPTD) transfer function model using the conventional relay autotune method. Usually, the value of the delay is assumed or noted from the initial portion of the response of the system. Whenever identifying a higher order dynamics system by a FOPTD model, this method identifies wrongly the time constant as negative (*Li* et. al., 1991) due to the error in identifying the time delay which is due to error in the model structures. In the present work, an additional equation is formulated to calculate accurately the parameters of the FOPTD model. Even when the actual system is FOPTD and the time delay to time constant ratio is larger, then higher order harmonics can not be neglected in the output response. Also in the present work, a method is proposed to consider the higher order harmonics of the relay oscillations. Improved accurate values for the controller ultimate gain are obtained. The estimated model parameters of a FOPTD model are compared with that obtained by *Li* et al. (1991) and that with the exact model parameters of FOPTD systems. The performance of the controller designed on identified model is compared with that identified by *Li* et al. (1991). The proposed method gives a better performance.

Keywords:
Relay tuning, higher order harmonics, FOPTD, single test

Introduction

Astrom and *Hagglund* (1984) have suggested the use of an ideal (on-off) relay to generate a sustained oscillation in the closed loop. The amplitude (A) and the period of oscillation (p_u) are noted from the sustained oscillation. The controller ultimate gain is calculated from $k_u = 4h/(\pi A)$, where h is the relay height and '*A*' is the process output amplitude. If the relay output is expanded in a Fourier series then it is possible that there is a considerable power present in the higher order harmonics. In deriving the equation for the ultimate gain of the controller, it is assumed that higher order harmonics of the relay output are filtered by the system and allows only the sine wave with the fundamental frequency of oscillation (*Astrom* and *Hagglund*, 1984). If the system has a low pass filter characteristics at the ultimate frequency, then the assumption is valid and the method gives an accurate estimate of the ultimate gain.

Luyben (1987) has used the relay feedback method to identify the model parameters (k_p, τ and D) of a first order plus time delay model. Using controller ultimate gain and the period of oscillation, two equations are formulated using the amplitude criterion and phase angle criterion. The value

of the time delay is noted from the initial portion of the relay output. The values of k_p and τ are then calculated from the derived two equations. For identifying FOPTD model for a system whose dynamics are of really higher order, then the method of Luyben gives negative time constant (*Li* et al., 1991). The error lies in the difference in the model structures.

Further when the ratio of time delay to time constant is larger, Li et al. (1991) have reported that the model identified by relay auto tune method gives as high as 27 % error in the value of k_u. For smaller D/τ, *Li* et. al. (1991) have found by simulation that the relay auto tune method gives –18 % error in the value of k_u. The auto tune method uses only principle harmonics for the calculation of model parameters. *Li* et al. (1991) and *Leva* (1994) have proposed the use of two relay feedback test (one conventional relay test and the second with added known delay). In these methods all the parameters of the model are identified. *Shen* et al. (1996) have used a biased relay for getting the model parameters using a single relay test. *Huang* et al. (1996) have proposed an asymmetrical relay method. *Yu* (1999) has given an excellent survey of relay feedback method. *Park* et. al.(1997) have proposed modifications of the method under load disturbances. Consideration of higher order dynamics is given by *Chiang* et. al. (1992), *Scali* et. al. (1999), *Luyben* (2001), *Kaya* and *Atherton* (1999)

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and *Atherton* (2000). *Scali* et. al. (1999) have proposed the use of several relay tuning tests to be carried out. *Luyben* method (2001) is not effective for unstable systems. In the methods proposed by *Atherton* (2000) and *Kaya* and *Atherton* (1999), computational complexity are involved. In the present work, (i) a modification in analyzing the data of the conventional relay method is proposed by formulating an additional equation so as to estimate more accurate value of the process gain k_p and hence more accurate FOPTD model parameters and (ii) a method to consider higher order harmonics in analyzing the conventional relay testing, for getting improved model parameters, is also proposed.

Estimation of Time delay

Recently Padmasree and Chidambaram (2001) have proposed a method for formulating an equation for time delay of the FOPTD model when analyzing the closed loop response of a PID controlled system. In the present work, their method in formulating the additional equation is applied here for analyzing the relay feedback oscillation.

Let $G(s)$ represent the process transfer function of the system

$$G(s) = y(s)/u(s) \tag{1}$$

From the definition of Laplace transform we get:

$$y(s) = \int y(t) \exp(-s\,t)\,dt \tag{2}$$

Let us evaluate the above integral (from $t = 0$ to ∞) for $s_1 = 8/t_s$, where t_s is the time at which few (say 3) repeated cycles of oscillation appear in the output. The reason for taking $s_1 = 8/t_s$ is that for $t > t_s$, because of very small value of the term [exp $(-s\,t)$], all further contribution by the subsequent terms is negligible while evaluating the integral value. Now we have the numerical value for $y(s_1)$. Similarly from the $u(t)$ versus time data, $u(s_1)$ is calculated. From the assumed FOPTD model also, we can formulate the equation for $y(s_1)$ as:

$$y(s_1) = u(s_1)\,k_p \exp(-D\,s_1)\,/(\tau\,s_1 + 1) \tag{3}$$

Thus we are able to formulate the above additional equation using $y(s_1)$ and $u(s_1)$. It is to be noted that we have to record the transient data of the relay till 2 or 3 sustained oscillations are obtained in the process output. From the amplitude criterion and phase angle criterion, we get the following equations for the parameters of FOPTD model as:

$$\tau = v/\omega_u \tag{4}$$

$$D = [\pi - \tan^{-1}(v)]/\omega_u \tag{5}$$

Using the above two equations in eq(3) we get:

$$k_p \exp[(-s_1/\omega_u)\{\pi - \tan^{-1}(v)\}]/[(v\,s_1/\omega_u) + 1] =$$
$$= y(s_1)/u(s_1) \tag{6}$$

where $v = [(k_u k_p)^2 - 1]^{0.5}$ and k_u is the controller ultimate gain.

Using the values of $y(s_1)$ and $u(s_1)$ in the above equations and solving the resulting nonlinear algebraic equation we get the value of k_p. Using the value of k_p in Eqs(4) and (5), we get the values of the FOPTD model parameters τ and D. Let us consider an example given by *Li* et al. (1991)

$$y(s)/u(s) = \exp(-2s)/[(10s + 1)(s + 1)] \tag{7}$$

Using the relay feedback method (with relay height as 1), we get the oscillations as shown in Fig 1. The steady oscillations show a pure sinusoidal wave form, thereby assumption of relay method to neglect higher order harmonics is valid for this example. By the *Luyben* (1987) method, the delay is noted as 2 from the initial response and from the oscillations it is noted that $\omega_u = 0.5978$ and $a = 0.1943$. Using the relation $k_u = 4h/(\pi A)$, we get $k_u = 6.553$.
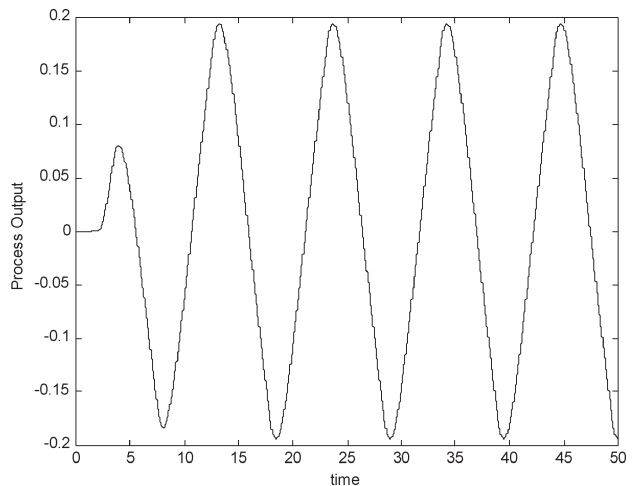

F i g .   1 – *Relay feedback oscillations for the system exp(–2s)/[(10s + 1)(s + 1)] relay height = 1*

Using the amplitude criterion and the phase angle criterion for a FOPTD system, *Li* et. al. (1991) have reported the values of k_p as –0.501 and τ as –5.03. Since negative values for τ and k_p are obtained, it is suggested by *Li* et al. (1991) that FOPTD model is not valid for this system and *Li* et al. (1991) fitted a second order plus time delay (SOPTD) model using a second relay test with a known additional delay time.

By the proposed method of analyzing the conventional relay method, we note from Fig 1 the

value of t_s as 68.9 and hence using Eq (2) we get $y(s_1) = -0.1239$ for the calculated value of $u(s_1) = -0.0407$. Using Eqs (6), (4) and (5), we get $k_p = 1.08$, $\tau = 11.725$ and $D = 2.87$. The proposed method is able to identify a FOPTD model. The open loop response of the actual system and the identified FOPTD model is shown in Fig 2. A good matching is obtained. Fig 2 shows that the response of the identified FOPTD model by the present method is better than that of SOPTD model proposed by *Li* et. al. (1991). Thus by properly interpreting the conventional relay oscillations, we can estimate the model parameters of a FOPTD model.
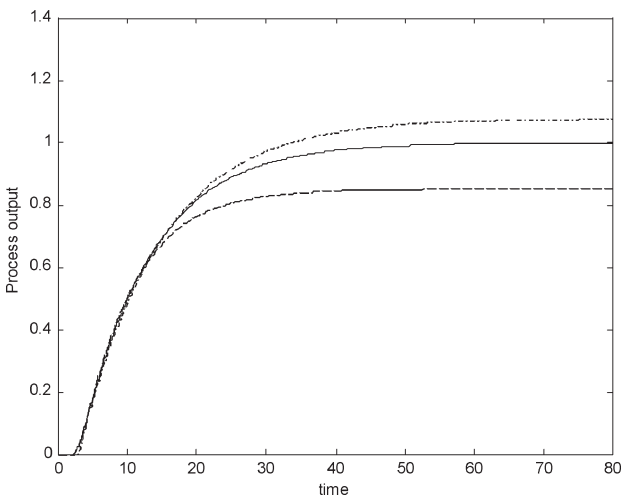

F i g . 2 – *The open loop comparisons of the original system and the identified model; middle line: actual system, upper line: FOPTD model by the present method, lower line: SOPTD model by Li et. al. (1991)*

Proposed method for considering higher order harmonics

From the Fourier series analysis, it can be easily shown (*Astrom* and *Hagglund*, 1984) that a relay consists of many sinusoidal waves of odd multiples of fundamental frequency 'ω' and with the amplitude $4h/(n\pi)$ ($n = 1,3,5,...$). The input to the process thus consists of many sine waves. For a FOPTD system, output wave is also a sinusoidal wave with different amplitude and different frequency. The oscillations what we observe i.e., $y(t)$ is the additional of many of sine waves:

$$y(t) = [A_1 \sin(\omega_u t + \phi_1) + (1/3)\, A_3 \sin(3\omega_u t + \phi_3) +$$

$$(1/5)\, A_5 \sin(5\omega_u t + \phi_5) + ...] \qquad (8)$$

where

$$A_1 = 1/[1 + (\tau\omega_u)^2]; \quad A_3 = 1/[1 + (3\tau\omega_u)^2];$$

$$A_5 = 1/[1 + (5\tau\omega_u)^2], ... \text{ etc.} \qquad (9)$$

$$\phi_1 = -D\omega_u - \tan^{-1}(\tau\omega_u) = -\pi \qquad (10a)$$

$$\phi_3 = -3D\omega_u - \tan^{-1}(3\tau\omega_u);$$

$$\phi_j = -j\, D\omega_u - \tan^{-1}(j\tau\omega_u) \qquad (10b)$$

$$y(t) = A_1\, [\sin(\omega_u t + \phi_1) + (1/3)\, b_3 \sin(3\omega_u t + \phi_3) +$$

$$(1/5)\, b_5 \sin(5\omega_u t + \phi_5) + ...] \qquad (11)$$

where

$$b_3 = \{[1 + (\tau\omega_u)^2]/[1 + (3\tau\omega_u)^2]\}^{0.5};$$

$$b_5 = \{[1 + (\tau\omega_u)^2]/[1 + (5\tau\omega_u)^2]\}^{0.5}; \text{ etc.}, (12)$$

If $\tau\omega_u$ is assumed very large (∞), then $y(t)$ will consists of only the fundamental harmonics. In Eq (8), the terms containing $A_3 \sin(3\tau\omega_u + \phi_3)$, $A_5 \sin(5\tau\omega_u + \phi_5)$ etc. will be neglected. As stated earlier, this assumption gives a large error in the calculation of k_u and hence in the estimated model parameters of FOPTD model. In what follows we will consider higher order dynamics for the calculation of the model parameters.

Let us derive approximate evaluation of $y(t)$ for the limiting cases of smaller $\tau\omega_u$ and separately for larger $\tau\omega_u$. Whenever the relay oscillation is close to rectangular wave form, the results for smaller $\tau\omega_u$ is to be used. If the relay oscillation is closer to triangular wave form, then the following result for larger $\tau\omega_u$ is to be used. If the relay oscillation is close to sine wave form, then the standard equation considering only the fundamental frequency of oscillation can be used.

case 1: when $\tau\omega_u$ is smaller

$$\phi_3 = -3D\omega_u - \tan^{-1}(3\tau\omega_u) \qquad (13a)$$

$$= -3D\omega_u - 3\tan^{-1}(\tau\omega_u) = 3\phi_1 = -3\pi \quad (13b)$$

Similarly,

$$\phi_5 = -5\pi; \ \phi_N = -N\pi \qquad (13c)$$

Hence Eq(11) can be written as:

$$y(t) = A_1[-\sin(\omega_u t) - (1/3)\, b_3 \sin(3\omega_u t) -$$

$$(1/5)\, b_5 \sin(5\omega_u t) + ...] \qquad (14)$$

For smaller value of $\tau\omega$, $\tau\omega_u$ can be neglected when compared to 1 and hence the values of b_1, b_3, ... b_N can each be approximated to 1. Hence Eq. (14) becomes:

$$y(t) = A_1[-\sin(\omega_u t) - (1/3)\sin(3\omega_u t) -$$

$$(1/5)\sin(5\omega_u t) + ...] \qquad (15)$$

Here the value of 'A_1' is to be calculated. The value of 'A_1' is not the amplitude what is observed from the output oscillation.

Li et al. (1991) have assumed that the system is a low pass filter so that higher order harmonics of the input sine waves are filtered and the system oscillation contains only the fundamental frequency. The value of k_u is obtained by *Li* et. al. (1991) from

$$k_u = 4h/(A_0 \pi) \qquad (16)$$

where A_0 is the observed output amplitude. In Eq (15), when only the first term is considered then $A = A_o$. Since $y(t)$ contains many sine waves and can not be represented by single sine wave of amplitude of A_0, the evaluated k_u deviates from the actual value. *Li* et. al. (1991) have reported that an error of –18 % to 27 % is noted in the estimated value of k_u by using Eq (16).

This problem can be overcome as follows: From the output oscillations, it is possible to calculate $y(t)$ at any time 't'. 'ω_u' is the frequency of observed oscillations. From Eq (15), we get

$$A_1 = y(t) / \Sigma \; [\sin(i\omega_u t)/i] \qquad (17)$$

Let us consider the time (t^*) at which

$$\omega_u t^* = 0.5\pi. \qquad (18)$$

Then Eq (15) becomes:

$$y(t^*) = A \; [1 - (1/3) + (1/5) -$$

$$(1/7) + (1/9) - \dots .] \qquad (19)$$

where $A =$ modulus of A_1. Let the number of terms to be considered in the above equation is denoted as N. Using the limiting value for the summation term (0.25π), we get from Eq (19):

$$A = 1.273 \; y(t^*) \qquad (20)$$

The time at which the value of $y(t^*)$ is to be noted from the response is given by

$$t^* = 0.5 \; \pi/\omega_u \qquad (21)$$

From the relay oscillation test, we can note down the value of ω_u and t^* can be calculated and the value of $y(t^*)$ is noted from the process output. The value of k_u is given by

$$k_u = 4h/(A\pi) \qquad (22)$$

The method is tested on FOPTD systems with various values of D/τ (refer to Table 1, for model parameters) and the results on k_u by the present method given in Table 2a along with the results of *Li* et al. (1991). From the identified values of k_u and the ultimate frequency of oscillation, the parameters of a FOPTD are evaluated. Table 1 gives the values of identified model parameters compared with the actual model parameters for different case studies of larger delay to time constant ratio.

If the system allows all the frequencies of oscillations (in that case, the output oscillation is also a rectangular wave form), then the limiting case of $\pi/4$ can be used in the above analysis in order to estimate the model parameters. However, if the process filters out beyond certain frequencies, then the output will very much deviate from the rectangular wave form. In that case, we have to use an appropriate value of N in order to get accurate values of the FOPTD model parameters.

Eq (19) shows that the sum of terms in the bracket varies from 1 to 0.8 for $N = 1$ to ∞. Hence, an error of 1.27 times of actual value is obtained when $N = 1$ is used instead of $N = \infty$. The number of terms to be considered in Eq (19) is any one of 1,3,5,7,9,… and ∞. When the number of terms in Eq (19) is ∞, then Eq (20) is considered for 'A'. *Li* et. al. (1991) have used $N = 1$ (i.e., $A = A_o$). The present study shows that the value of $N = 5$ gives a better result on the calculated k_u and on the model

T a b l e 1 – *Comparison of estimated k_u and identified FOPTD model parameters for the present method (when $\omega t = \pi/2$) and Li et. al. (1991) method*

No	Simulated Model Parameters			Ultimate gains			Freq. From osc	Identified Model Parameters			
	k_p	τ	D	k_u (ana)	k_u (Li)	kD_u (Pro)	ω (osc)	k_p (Li)	k_p (Pro)	τ (Li)	τ (Pro)
1	1	0.2	4	1.01	1.27	1.00	0.76	0.79	1.00	0.14	0.14
2	1	0.4	4	1.04	1.27	1.068	0.74	0.80	0.98	0.28	0.42
3	1	0.5	4	1.06	1.27	1.077	0.72	0.82	0.97	0.37	0.45
4	1	1.0	4	1.19	1.30	1.18	0.67	0.86	0.96	0.74	0.81
5	1	1.0	5	1.13	1.28	1.13	0.55	0.83	0.96	0.73	0.78

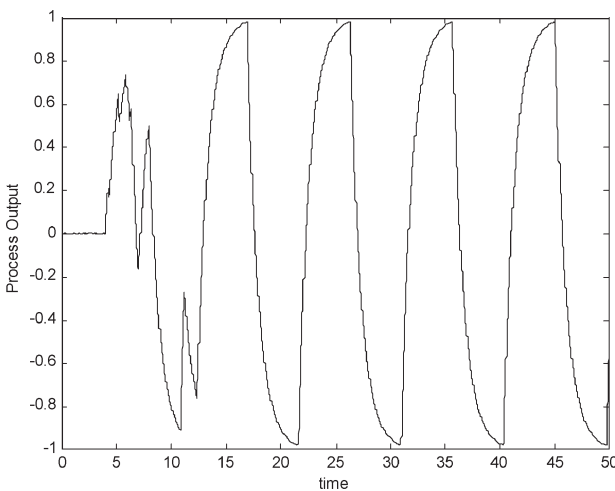The time delay values for the present method are 4.133, 3.83, 3.93, 3.94, 4.95 respectively for No 1 to 5.

T a b l e   2 a – *Effect of including the higher order harmonics on k_u (Eqs.19 & 22)*

D/τ	N = 1 k_u	N = 3 k_u	N = 5 k_u	N = 7 k_u	N = 9 k_u	N = ∞ k_u	k_u exact
4	1.30	1.220	1.175	1.155	1.145	1.106	1.19
5	1.28	1.172	1.128	1.109	1.098	1.061	1.13
8	1.27	1.118	1.077	1.059	1.050	1.013	1.06
10	1.27	1.109	1.068	1.051	1.040	1.005	1.04
20	1.27	1.10	1.06	1.04	1.03	1.02	1.01

T a b l e   2 b – *Details of calculation for Table 2a*

D/τ	A_0 observed amplitude	ω_u (frequency)	t^* (= $0.5\pi/\omega_u$)	$Y(t^*)$	A Calculated amplitude
4	0.9817	0.67	2.34	0.904	1.089
5	0.9933	0.69	2.28	0.942	1.127
8	0.9997	0.72	2.17	0.987	1.182
10	1.0000	0.74	2.14	0.995	1.225
20	1.0000	0.76	2.08	1.00	1.273

T a b l e   2 c – *Effect of including the higher order harmonics on k_u (use of Eqs. 33&22)*

D/τ	N = 1 k_u	N = 3 k_u	N = 5 k_u	N = 7 k_u	N = 9 k_u	N = ∞ k_u	k_u exact
0.2	7.023	8.73	8.98	9.09	9.15	9.36	8.50
0.08	16.64	19.78	20.34	20.58	20.72	21.21	20.28
0.04	32.39	38.06	39.15	39.61	39.88	40.82	39.98

T a b l e   2 d – *Details of calculation for Table 2c*

D/τ	A_0 observed amplitude	ω_u (frequency)	t^* (= $0.5\pi/\omega_u$)	$Y(t^*)$	A Calculated amplitude*
0.2	0.1813	0.4274	3.67	0.1678	0.1458
0.08	0.0765	0.408	3.85	0.0741	0.0644
0.04	0.0391	0.4002	3.925	0.0385	0.0334

* using first three terms in the series

parameters of FOPTD. Fig 3 shows the system oscillations (both the dynamics and steady-state) of relay test for the system $\exp(-4s)/(\tau s + 1)$ with $\tau = 1$ and 0.4 (i.e., $D/\tau = 4$ & 10). If the system's steady-state oscillation resembles that of pure sinusoidal wave (as shown in Fig 1), then a value of $N = 1$ can be used. If the oscillation deviates from pure sine wave and depending on the extent of deviation, $N = 3$ or 5 or 7 can be considered. For example for the Fig 3a, a value of $N = 3$ or 5 can be considered. When the oscillation is near to that of rect-
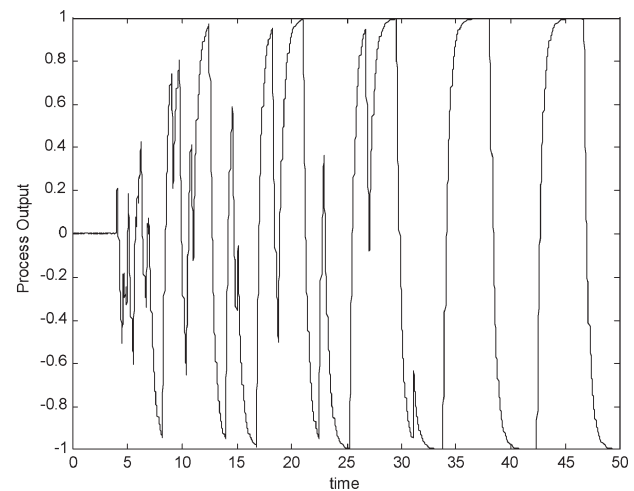
angular then a value of $N = 9$ can be considered (example for the condition of Fig 3b). An empirical relation between D/τ versus N is fitted as

$$N = -0.0395(D/\tau)^2 + 1.4746(D/\tau) - 1.9266 \quad (23)$$

The nearest integer value of N from the above equation is to be used

In the later section we will show, by a simulation study on a higher order system, the improved

(a)

(b)

F i g . 3   – *Relay feedback oscillations for the system (a) exp(–4s)/(s + 1), (b) exp(–4s)/(0.4s + 1) relay height = 1*

estimate of k_u and hence on the identified model parameters of a FOPTD system. Now let us consider the limiting case of larger $\tau\omega_u$ for calculation of k_u.

case 2: limiting case of larger $\tau\omega_u$

$$\phi_1 = -D\omega_u - \tan^{-1}(\tau\omega_u) = -\pi \qquad (24a)$$

$$= -D\omega_u - (\pi/2) = -\pi \qquad (24b)$$

hence

$$D\omega_u = 0.5\pi \qquad (25)$$

$$\phi_3 = -3D\omega_u - \tan^{-1}(3\tau\omega_u) \qquad (26a)$$

$$= -3(D\omega_u) - \pi/2 = -4\pi/2 \qquad (26b)$$

Similarly,

$$\phi_5 = -6\pi/2;\ldots;\ \phi_N = -(N+1)\pi/2 \qquad (27)$$

Hence Eq(11) can be written as:

$$y(t) = A_1[-\sin(\omega_u t) - (1/3)\ b_3\ \sin(3\omega_u t) -$$

$$(1/5)\ b_5\ \sin(5\omega_u t) + \ldots\ldots] \qquad (28)$$

where

$$b_3 = \{[1 + (\tau\omega_u)^2]/[1 + (3\tau\omega_u)^2]\}^{0.5};$$

$$b_5 = \{[1 + (\tau\omega_u)^2]/[1 + (5\tau\omega_u)^2]\}^{0.5}$$

For larger value of $\tau\omega$, 1 can be neglected when compared to the value of $\tau\omega_u$ and hence the values of b_1, b_3, .. can be approximated as:

$$b_3 = 1/3;\ b_5 = 1/5;\ b_j = 1/j,\ \text{etc.} \qquad (29)$$

Hence Eq (28) becomes:

$$y(t) = A_1[-\sin(\omega_u t) + (1/9)\ \sin(3\omega_u t) -$$

$$(1/25)\ \sin(5\omega_u t) + \ldots\ldots] \qquad (30)$$

Here the value of 'A_1' is to be calculated. The value of 'A_1' is not the amplitude what is observed from the output oscillation.

Li et. al. (1991) have reported that an error of -19 % is noted in the estimated value of k_u by using Eq (22) for larger values of $\tau\omega_u$. This problem can be overcome as follows: From the output oscillations, it is possible to calculate $y(t)$ at any time 't'. 'ω_u' is the frequency of observed oscillations. From Eq (30), we get

$$a = y(t)/\Sigma\ [\sin(i\omega_u t)/i^2] \qquad (31)$$

Let us consider the time (t^*) at which

$$\omega_u t^* = 0.5\pi. \qquad (32)$$

Then Eq (30) becomes:

$$y(t^*) = A\ [1 + (1/9) + (1/25) +$$

$$(1/49) + (1/81) + \ldots\ldots] \qquad (33)$$

Let the number of terms to be considered in the above equation denoted as N. Using the limiting value for the summation term $(0.125\pi^2)$, we get from Eq (33):

$$A = 0.810\ y(t^*) \qquad (34)$$

The time at which the value of $y(t^*)$ is to be noted from the response is given by

$$t^* = 0.5\ \pi/\omega_u \qquad (35)$$

Table (2b) compares the values of k_u from Eq (33) for different number of terms to be considered in Eq (33) for the lower values of D/τ. The proposed method of considering the higher order harmonics is able to explain the reported maximum error of –18 % and +27 % as shown in Tables (2b) and (2a) respectively.

Simulation study

Let us consider an example with a higher order dynamics:

$$y(s)/u(s) = 1\ \exp(-4s)/(0.5s + 1)^3 \qquad (36)$$

Using the relay feedback method (with relay height as 1) we get an oscillation shown in Fig 4. From Fig 4, we noted from the output ω_u as 0.5889 and 'A' as 0.9969. Fig 4 shows that the oscillations contain a higher order harmonics (since the steady oscillation is not a pure sinusoidal wave form). From Fig. 4, we note $t_s = 56$ and hence $s_1 = 8/56$. The value of $y(s_1)$ and $u(s_1)$ are calculated. 5 terms ($N = 5$) are used in evaluating the sum term of Eq (19). The calculated value of 'A' is 1.165. The identified value of $k_u = 1.093$. Hence, the present method gives the model parameters of FOPTD as $k_p = 0.9714$, $D = 4.7061$ and $\tau = 0.605$. *Li* et al. (1991) have reported the identified value of k_u as 1.28 and that of the
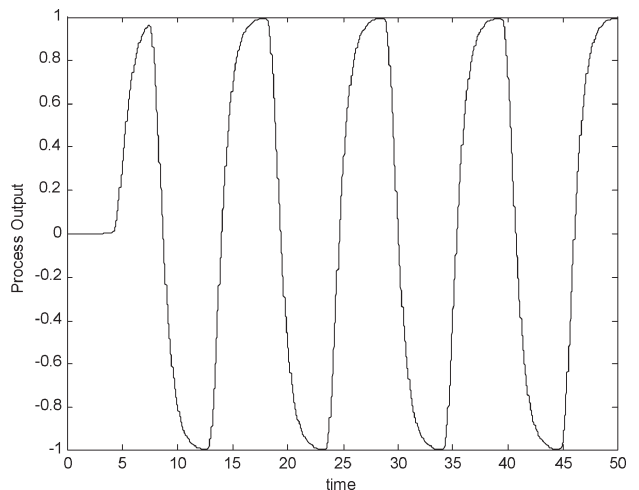

F i g . 4 – *Relay feedback oscillation for the system* $exp(-4s)/(0.5s + 1)^3$ *relay height = 1*

model parameters as $k_p = 1.0841$, $D = 4.04$ (noted from the initial response) and $\tau = 1.6362$. *Li* et al. (1991) assumed all higher order harmonics are filtered out by the system (Fig 4 shows that it is not so). The open loop response of the actual system and that of the identified FOPTD models are compared in the Fig 5. The response by the present model is better than that of *Li* et al. (1991). A PID controller is designed based on k_u and P_u by using Ziegler-Nichols continuous cycling tuning method. The PID settings by the present identified model are: $k_c = 0.656$; $\tau_I = 5.34$ and $\tau_D = 1.334$ and by *Li* et. al model as $k_c = 0.768$; $\tau_I = 5.34$ and $\tau_D = 1.335$. The performances of the PID controllers are evaluated on the original transfer function model of the process. The performance of the PID controller based on the present model gives an improved performance as shown in Fig 6.
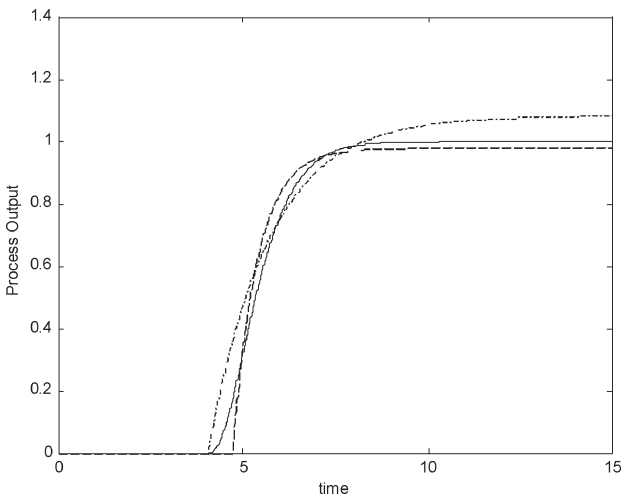

F i g . 5 – *Open loop comparisons of actual system and the identified FOPTD models; solid: actual system*
$exp(-4s)/(0.5s + 1)^3$, *dash: FOPTD model by present method, dot: FOPTD model by Li et. al. (1991) method*
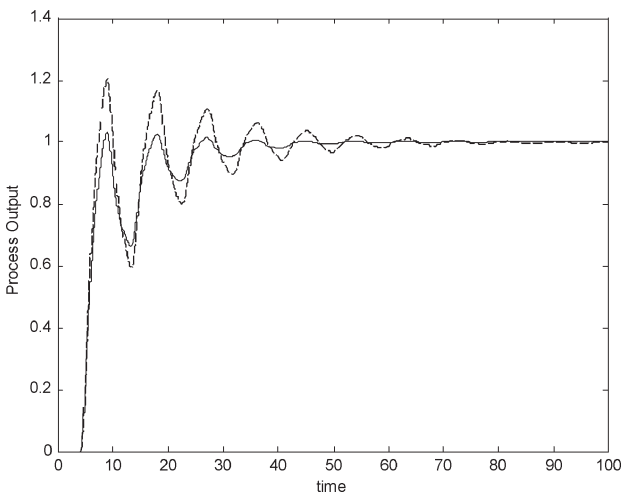

F i g . 6 – *The closed loop response of the system:*
$exp(-4s)/(0.5s + 1)^{0.5}$ *(controller design by closed loop Ziegler--Nichols method), solid: present method, dash: Li et al.*

PID controllers are also designed based on the FOPTD model parameters by using Ziegler-Nichols open loop tuning formula. Since the ratio of delay to time constant is more than 1, the appropriate modified Ziegler-Nichols tuning formulae (*Chidambaram*, 1998) are used: $k_c = 0.6462 [1 + (\pi\tau/D)^2]^{0.5}$; $\tau_I = D$ and $\tau_D = 0.25\ D$. The PID settings for the present identified model are $k_c = 0.73$; $\tau_I = 4.70$ and $\tau_D = 1.18$ whereas for the model identified by *Li* et. al. (1991) are $k_c = 0.971$; $\tau_I = 4.04$ and $\tau_D = 1.01$. Fig 7 shows the performance comparison of the closed loop system evaluated on the original higher order system using the PID settings based on identified model. The performance of the present method gives a better response than that identified by *Luyben* (1987) method.
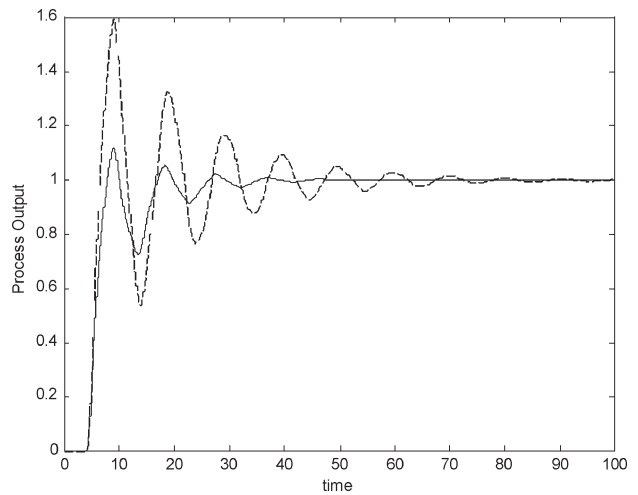

F i g . 7 – *The closed loop performance of the system:*
$exp(-4s)/(0.5s + 1)^{0.5}$ *(controller design by open loop Ziegler--Nichols method), solid: present method, dash: Li et al.*

The effect of measurement noise is studied by adding a random noise (with mean = 0 and the standard deviation = 0.5 %) to the process output and the corrupted signal is used for relay feedback. The present method gives $s_1 = 0.1344$, $\omega_u = 0.72$, $y_{s1} = 0.2168$, $u_{s1} = 0.3882$ and hence the identified model parameters of the FOPTD are obtained as $k_p = 1.005$, $D = 3.82$ and $\tau = 0.5$. These values are closer to the actual model parameters of $k_p = 1$, $D = 4.0$ and $\tau = 0.5$.

Conclusions

A method is suggested to formulate an additional equation so that the process gain can also be estimated using the conventional relay auto-tune method. This method avoids getting a negative time constant of a FOPTD model. For systems showing higher order harmonics in the response, a modifica-

tion of the calculation for the model parameters of FOPTD model using the conventional relay feed back method is also proposed. This method does not assume the complete filtering of higher order harmonics. The method of calculation is also simple. The present method gives an improved value for the controller ultimate gain. The method gives more accurate results [on k_u and on the identified FOPTD model parameters] than that proposed by *Luyben* (1987) and *Li* et al. (1991). Simulation results show that the present method gives improved open loop and as well as closed loop performances.

Nomenclature

A_o – amplitude of oscillation observed from the process output

A_1 – amplitude of oscillation corresponds to the principle harmonics calculated from Eq(22)

G – process transfer function

h – relay height

L – process delay

k_c – controller gain

k_p – process gain

k_u – controller ultimate gain

N – number of terms considered in Eq (19) or Eq (33)

p_u – period of output oscillation

s – Laplace variable

s_1 – $= 8/t_s$

t_s – time taken to reach 3 invariant cycle of oscillation in the output

t – time

t^* – $= 0.5\ \pi/\omega_u$

u – input variable

v – $= [(k_u k_p)^2 - 1]^{0.5}$

y – output variable

τ – process time constant

τ_d – process time delay

τ_I – integral time

τ_D – derivative time

ω – frequency of oscillation

ω_u – ultimate frequency of oscillation

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