Measurement and Evaluation of Drag Coefficient for Settling of Spherical Particles in Pseudoplastic Fluids*

G. Matijašić and A. Glasnović
University of Zagreb, Faculty of Chemical Engineering and Technology
Marulićev trg 19, 10 000 Zagreb, Croatia, e-mail: gmatijas@pierre.fkit.hr

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The drag coefficient is an important hydrodynamic characteristic of the motion of particles in both Newtonian and non-Newtonian fluids and the possibility of its evaluation based on mathematical equations is very significant. Development of mathematical model for estimating drag coefficient values is primarily based on measurement accuracy.

The experiments were run using $w = 1 - 4$ % carboxymethylcellulose (CMC) aqueous solutions with different rheological properties which were determined by viscosity measurements. Approximately 200 experimental results were obtained using spherical particles of different materials and diameters. Measurements of its falling velocities were carried out in a glass tube. An optical method of measurement was developed for this purpose. The laboratory device contains three printed boards. Two are identical and each consists of 7 photodiode/photoreceiver pairs, while the third is used for connecting with a computer. Photodiodes emit 2 mm wide IR rays towards photoreceivers and both have TTL levels as output. Output signals from these two boards are connected with RS flip-flop on the third board, and its output is connected to the computer. This technique enables time measurement accuracy of 0.02 s.

By comparison of experimental drag coefficient values with those obtained using proposed model for pseudoplastic fluids, the value of mean relative deviation is 25 %. Our proposed mathematical model simplifies the correlation of correction factor vs. flow behaviour index and achieves better results for a wide range of Reynolds number ($Re_{PS} <$ 1000) giving 15 % mean relative deviation.

Keywords:
Drag coefficient, power law fluid, optical method

Introduction

Modelling of many industrial processes is based on fundamentals of particles falling in fluids. The most important hydrodynamic characteristic is drag coefficient C_D. Falling of spherical and non spherical particles in Newtonian and non-Newtonian fluids and evaluation of drag coefficients have been studied by many authors[1,2,3,4] who proposed a large number of models for evaluation of drag coefficients. This work contains comparison of experimental drag coefficient values with those obtained using model proposed by *Kawase* and *Ulbrecht*[1,2,3] which is based on model proposed by *Acharya*[4]. The suitability of proposed model was evaluated by calculating values of mean relative deviations.

Experimental

Laboratory device contains three printed boards placed on a 5.4 cm internal diameter glass tube at 0.5 m distance from each other, as shown in Fig. 1, and one computer.

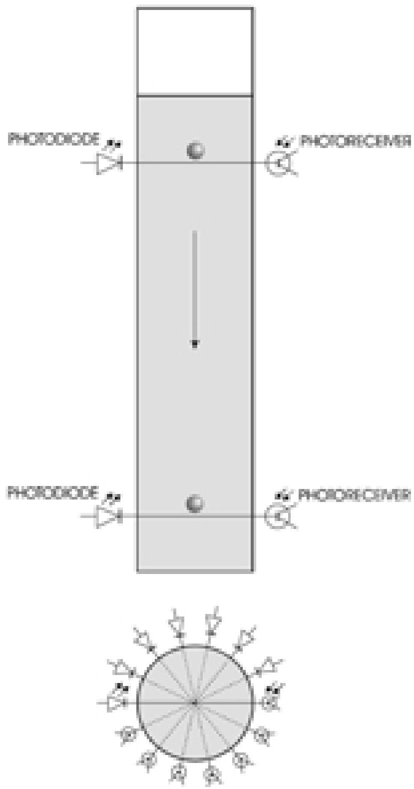

Fig. 1 – *Laboratory device for time measurement of spherical particles falling in fluids*

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Two printed boards are used for position detection of spherical particles. They are identical and each consists of 7 photodiode/photoreceiver pairs. The third is used for connecting with the computer. Photodiodes emit 2 mm wide IR (Infrared) ray by turns towards photoreceivers whose outputs give TTL (Transistor-Transistor Logic) levels. Output signals from these two boards are connected with RS flip-flop on the third board. Passing of spherical particle beside upper board interrupts IR ray and sets an input of RS flip-flop to "1", until interruption of IR ray at down board resets RS (Reset Set) flip-flop to the original value. This means that input of RS flip-flop is set again to "0". RS flip-flop output signal is connected to the computer, which enables direct time measurement with 0.02 s accuracy.

This measurement technique is suitable only for transparent fluids since photoreceiver can not detect IR rays in opaque fluids. It was also found that higher viscosity samples have difficulties releasing air bubbles that appear when filling a tube, which also causes IR rays interruption. Therefore, it is necessary to leave those samples in a tube for at least 24 hours before experiments.

The experiments were run using carboxymethylcellulose aqueous solutions in fraction range $w = 1 - 4$ % with different rheological parameters. Rheological behaviour of used solutions can be described with power-law model:

$$\tau = K \cdot \dot{\gamma}^{n} \tag{1}$$

where K is consistency index, and n flow behaviour index. Rheological parameters were determined using RV-3 HAAKE Viscometer and are shown in Table 1.

Table 1 – *Rheological parameters and density of used CMC solutions at working temperatures*

w_{CMC}/%	K/Pa s^n	n	ρ/kg m^{-3}	T/°C
1,00	0,023	0,948	1004	25
1,50	0,105	0,866	1008	25
2,00	0,617	0,794	1011	25
2,25	0,693	0,744	1013	20
2,50	1,353	0,688	1015	25
3,00	1,370	0,639	1018	26
3,20	1,630	0,576	1020	29
3,35	0,826	0,694	1021	25
3,50	3,203	0,610	1022	26
3,70	3,784	0,595	1023	20
3,85	5,127	0,596	1025	24
4,00	7,384	0,537	1026	20

Since the rheological parameters of carboxymethylcellulose (CMC) solutions are strongly influenced by solution temperature, rheological parameters were determined at same temperature as measurements.

Spherical particles of different materials and different diameters were used (Table 2).

Table 2 – *Particle densities and diameters used in experiment*

Material	ρ_s/kg m^{-3}	d/cm
steel	7 740 – 7 948	0,346 – 2,060
glass	2 431 – 2 553	1,496 – 2,492
plumb	11 008	0,692

Combination of different solutions and different particles resulted in 200 experimental results.

Results and discussion

Considering spherical particles falling in fluids, Newton's law can be used for evaluation of drag coefficient in laminar, transition and turbulent region:

$$C_D = \frac{4}{3} \cdot \frac{(\rho_s - \rho) \cdot g \cdot d}{\rho \cdot u^2} \tag{2}$$

Those values were corrected for the value of wall effect[5].

Particle Reynolds number was calculated using equation (3) modified for power-law (pseudoplastic) fluids:

$$Re_{PS} = \frac{u^{2-n} \cdot d^n \cdot \rho}{K} \tag{3}$$

Experimental results were compared with those obtained by empirical model that is suitable for evaluation of drag coefficient for pseudoplastic fluids with $0{,}5 \leq n < 1$. In the laminar region, for $Re_{PS} < 1$, the values of drag coefficient were calculated using[1]

$$C_D = \frac{24}{Re_{PS}} \cdot X(n) \tag{4}$$

and in the transition region, for $1 < Re_{PS} < 500$, using

$$C_D = \frac{24}{Re_{PS}} \cdot X(n) + \frac{10.5 \cdot n - 3.5}{Re_{PS}^{(0.32 \cdot n + 0.13)}} . \tag{5}$$

where $X(n)$ is the drag coefficient correction factor and is given by equation (6)[1].

$$X(n) = 3^{(3\cdot n-3)/2} \cdot \frac{-22\cdot n^2 + 29\cdot n + 2}{n\cdot(n+2)\cdot(2\cdot n+1)} \quad (6)$$

The mean relative deviation was used to evaluate the agreement of experimental results and empirical model[6]:

$$\overline{\delta} = \frac{1}{N}\cdot\sum_{i=1}^{N}\frac{\left|C_{\mathrm{D,exp},i} - C_{\mathrm{D,teor},i}\right|}{C_{\mathrm{D,exp},i}}\cdot 100 \quad (7)$$

Experimental results were compared with those obtained by the semiempirical model proposed by *Kawase* and *Ulbrecht* (Fig. 2). It is evident that the model is useful at lower Reynolds numbers ($Re_{PS} < 200$) while at higher values deviation is significant. Value of mean relative deviation for this model is 25 %, considering all the experimental runs.

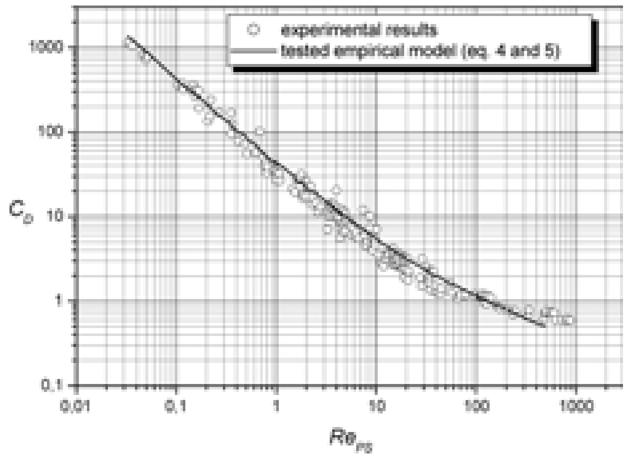

Fig. 2 – *Experimental and semiempirical model*(1) *values of drag coefficient vs. Reynolds number*

It was necessary, in order to reduce value of mean relative deviation, to find a model that better correlates with experimental results at higher values of Reynolds numbers. Mathematical regression of experimental data resulted with simplification of the drag coefficient correction factor:

$$C_{\mathrm{D}} = \frac{24}{Re_{\mathrm{PS}}}\cdot A(n) + 0.653 \quad (8)$$
$$A(n) = -1.26\cdot n + 2.3$$

The obtained empirical correlation is applicable in wider range of Reynolds number ($Re_{PS} < 1000$) and the mean relative deviation is 15 %. Comparison of experimental results and our proposed model can be seen in Fig. 3.

For final confirmation of reliability of this model it is still necessary to make further analysis and more experiments in wider range of Reynolds numbers.

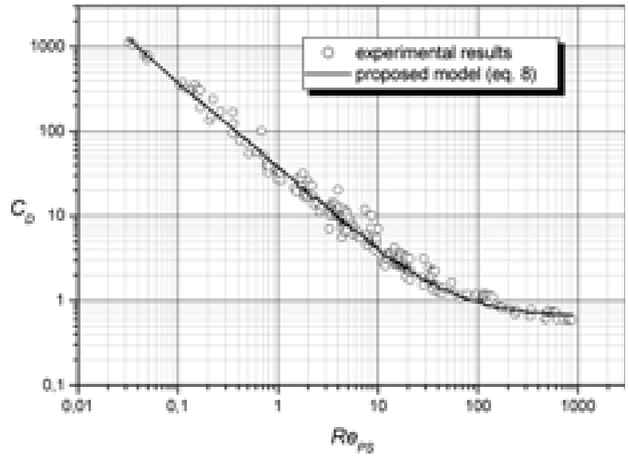

Fig. 3 – *Experimental and proposed model values of drag coefficient vs. Reynolds number*

Conclusions

A measurement technique was developed enabling time measurement of spheres falling in fluids with great accuracy (0.02 s). The disadvantage of this method is that it can not be used for measurements in opaque fluids.

A proposed mathematical model (eq. 8) that includes a new drag coefficient correction factor enables more precise evaluation of drag coefficients, than previously proposed semiempirical model of Kawase and Ulbrecht, for used type of polymer solution (CMC).

Symbols

A(n) – proposed drag coefficient correction factor (eq. 8)
C_{D} – drag coefficient
d – particle diameter, m
$\dot{\gamma}$ – shear rate, s^{-1}
$\overline{\delta}$ – mean relative deviation, %
g – gravitational acceleration, m s^{-2}
K – consistency index for pseudoplastic fluids, Pa s^n
n – flow behaviour index for pseudoplastic fluids
N – number of experiments
Re_{PS} – particle Reynolds number for pseudoplastic fluids
ρ – solution density, kg m^{-3}
ρ_{s} – particle density, kg m^{-3}
T – temperature, °C
τ – shear stress, Pa
u – particle falling velocity, m s^{-1}
w – mass fraction, %
X(n) – drag coefficient correction factor, a function of *n* (eq. 6)

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