Experiment-based Comparison of Prediction Methods for Pump Head Degradation with Viscous and Power-law Fluids

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Although several methods are known to calculate pump performance with highly viscous and non-Newtonian fluids, research has not yet determined all the key parameters of these predictions. It is unclear how these parameters depend on the pump geometry and the delivered fluid rheology, which can vary widely in the chemical industry. In our study, the performance curves of a radial centrifugal pump with a viscous Newtonian glycerol solution and a non-Newtonian power-law fluid were experimentally compared. The head degradation of the pump was also presumed with the ANSI/HI and the Ofuchi methods, which are evident and commonly used for viscous Newtonian fluids, but not for non-Newtonians. The required constants were estimated based on experimental data for both models, and the Ofuchi method was adapted to power-law fluid. Based on our results, the Ofuchi method proved to apply for head degradation prediction with the examined power-law fluid.

Keywords:
centrifugal pump, experiments, head degradation, performance curves, power-law fluid, viscous fluid

Introduction

There are many applications in various fields of industry where highly viscous and non-Newtonian fluids are delivered by pumping. Some of them can be characterised with power-law model[1], for instance, juices[2,3] and liquid egg yolk[4,5] in the food industry, crude oils in the petroleum industry[6], fresh concrete in the materials industry[7], activated sludge in wastewater treatment[8], and polymer solutions[9] in the chemical industry. Moreover, the rheology of the fluid can also change during the technological process[10,11], which influences the system's operation and efficiency.

When pumping highly viscous and non-Newtonian materials, the pipeline system's curve and the pump's performance curves are also required for providing energy-efficient and safe operation. It is known that the system's characteristic curve can change because the pressure losses of its elements with power-law fluids differ from those with Newtonian fluids[2,12–15]. The manufacturer gives the baseline performance curves of the centrifugal pump with water, i.e., the head-volume flow rate and efficiency-volume flow rate curves. A question arises of how the non-Newtonian rheology of the delivered fluid changes (degrades) the performance curves of the pump.

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There are two groups of methods in the literature for estimating the curve alterations for a highly viscous Newtonian fluid. Both methods predict correction factors, by which the points of the original performance curve can be modified to obtain the degraded characteristic curve point-by-point. The most commonly used methods are available for pumping viscous fluids; some are modified to apply to non-Newtonian materials.

The first group of the known methods works with experimentally determined factors. The American standard (American National Standard Institute/Hydraulic Institute – ANSI/HI) was made for highly viscous materials[16]. The relationships required to calculate the correction factors in this method (referred to as ANSI/HI method) are based on the determination of parameter B, which includes the kinematic viscosity of the fluid. In addition, the data of the Best Efficiency Point (BEP) are required for the calculations. In the same group, the KSB method is also empirical and feasible for viscous materials. The pump manufacturing company developed a simple graphical method[17], and it is widely used in engineering practice.

Recently, one of the leading research directions has been adapting the ANSI/HI method to the non-Newtonian material properties. The difficulty of the work lies in the fact that, while the calculation formulae contain one viscosity value, the viscosity is a function of the shear rate for non-Newtonian materials, thus using apparent viscosity value is recommended[18]. The relevant shear rate can be different at each point of the performance curve. On the other hand, determining an average value of the viscosity is not trivial, as the shear rate changes over a wide range inside the pump[19].

Walker and Goulas compared the ANSI/HI method with experiments for Bingham plastic fluids[20]. They determined the apparent viscosity and the pump Reynolds number for the shear rates corresponding to 2 · *n*, where *n*, was the rotational speed. Sery *et al.*[21] refined their method: a mean shear rate was estimated in the pump impeller according to Metzner and Otto[22]. In their work, the correlation between the shear rate and the pump's rotational speed was experimentally determined with a Herschel-Bulkley fluid. Pullum and Graham[23] stated that the shear rate is not constant inside the impeller. Therefore, an equivalent pipe diameter was introduced to substitute the pump. This quantity is a function of the impeller diameter and parameter *w*. For a given volume flow rate with the equivalent diameter, the characteristic velocity can be calculated. The shear rate can be expressed with the Rabinowitsch-Mooney equation[24] in the laminar case. In the case of turbulent flow, the apparent viscosity value for the shear rate of 4000 s^{-1} is recommended.

The limitation of their method is choosing the value of *w* to determine the equivalent pipe diameter. Pullum and Graham suggest 25 % of the impeller diameter[23]. Furlan *et al.* proposed 13.7 % and 15.8 % impeller diameter for two different pumps they had tested[25]. By experiments, other researchers found an up to 3.1–37.5 % span in the value of *w* parameter for a given type of pump, varying with the fluid[26]. Kalombo *et al.* concluded that it is impossible to predict *w* analytically so that it should be determined empirically for each pump with a new fluid[27].

The second group of methods for calculating performance curves for highly viscous materials is based on dimensional analysis. Stepanoff showed that the introduction of dimensionless numbers provides a tool for calculating the correction factors[28]. The dimensionless groups that characterise a pump are the pressure number $\psi = 2gH/(\pi^2 n^2 D_2^2)$, flow number $\phi = 4Q/\left(D_2^3 \pi^2 n\right)$, Reynolds number $Re = nD_2^2/\nu$, where g [m s^{-2}] is the gravitational acceleration, H [m] is the head, n [s^{-1}] is the rotational speed, D_2 [m] is the outlet diameter of the impeller, Q [m^3 s^{-1}] is the volume flow rate, and ν [m^2 s^{-1}] is the kinematic viscosity of the fluid. Using the data of the BEP, these values can be normalised not to include the pump geometry. A simple correlation between the head correction factor and the volume flow rate correction factor calculated for a given normalised specific speed was presented[28,29]. The relationship between the head correction factor and the announced modified Reynolds number was estimated from experimental results by Gülich *et al.*[30] and Ofuchi *et al.*[31] Ofuchi suggested a new model predicting head degradation with highly viscous fluids[31]. Although Csizmadia *et al.* showed that this method (referred to as the Ofuchi method) could be useful with one power-law fluid[32], the dependence of the evaluating constants on pump type, impeller geometry, and material properties is still an open question. Furthermore, research to date has not yet revealed how to convert this method for other non-Newtonian fluids.

Our main goal was to predict and experimentally verify the head degradation of a centrifugal pump with the ANSI/HI and the Ofuchi method with viscous Newtonian and a power-law test fluid.

Experiments

Rheology

Two test fluids were investigated: a Newtonian 25 % glycerol solution and a non-Newtonian jelly textured bath gel (referred to as 'gel'). An Anton Paar Physica MCR301 rotational viscometer was used to determine the rheological parameters of the test fluids, with thermoelectric temperature control via built-in Peltier elements. The measurement range was 0.1 – 100 s^{-1}. The instrument was used with a cone-plate layout with a gap of 0.054 mm. Both liquids were measured three times, and the parameters of the rheograms were determined based on their mean. The gel was evaluated with a power-law rheology model and showed pseudoplastic behaviour. The fit is presented in Fig. 1. The describing equations of the test fluids were:

$$\tau_{\text{glycerol}} = 0.00180 \cdot \dot{\gamma} \tag{1}$$

$$\tau_{\text{gel}} = 4.796 \cdot \dot{\gamma}^{0.3127} \tag{2}$$

where τ [Pa] is the shear stress, and $\dot{\gamma}$ [s^{-1}] is the shear rate of the fluid. The density values of the fluids were: ρ_{water} = 1000 kg m^{-3}; ρ_{glycerol} = 1068 kg m^{-3}; ρ_{gel} = 1010 kg m^{-3}.

Equipment

A two-stage radial centrifugal pump, type Wilo-Helix-EXCEL 1602-1/16/E/KS, was built in a test rig. The specific speed $n_q = n_n Q_{\text{BEP}}^{0.5} \left(H_{\text{BEP}}/z\right)^{-0.75}$ characterised the shape of the turbomachine impeller, where n_n [rpm] is the nominal rotational speed, Q_{BEP} [m^3 s^{-1}] and H_{BEP} [m] are the flow rate and the head in the BEP point, and z is the number of the

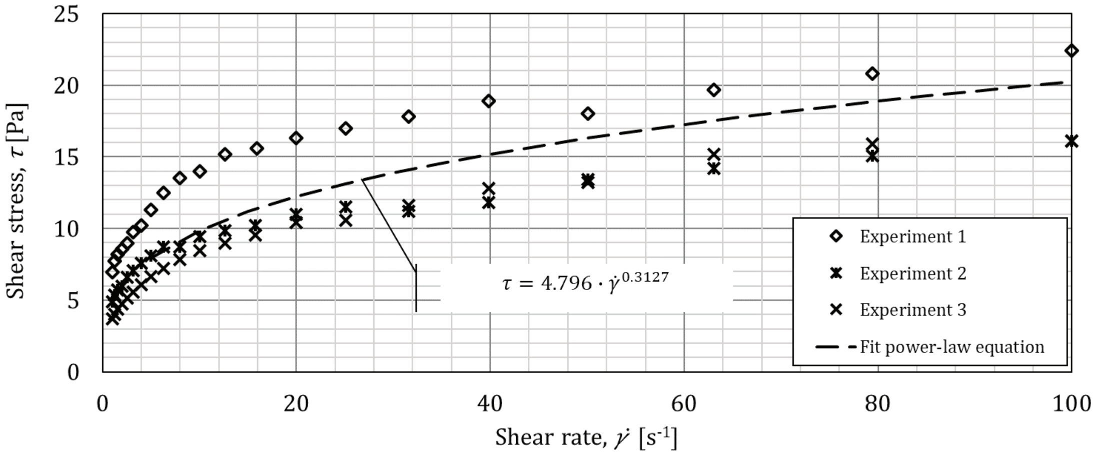

Fig. 1 – *Measured and fit rheology curves of the gel*

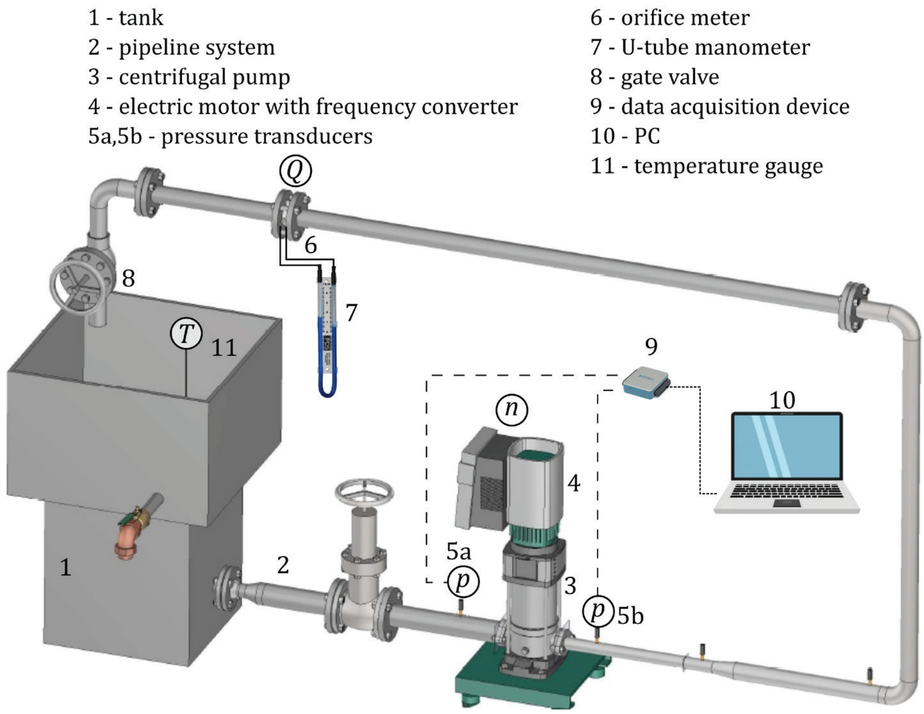

Fig. 2 – *Experimental setup*

stages[2]. For our pump they were $n_q = 35$ and $z = 2$; the impellers diameter were $D_{imp} = 0.1$ m. The actual rotational speed (n [rpm]) was modified using the frequency converter, and the operation points were adjusted with a gate valve at the pressure side of the system. The static head (H [m]) was calculated from the pressure difference measured with calibrated pressure transducers at the pressure and suction side of the pump. The pressure losses of the additional straight pipe sections were subtracted from the pressure difference. The volume flow rate (Q [m^3 s^{-1}]) was determined with a standard orifice meter. This method was also checked with a Fuji Electric (FSS-C1YY1) ultrasonic flowmeter and bucketing. The temperature of the fluid was checked with a temperature gauge in the tank. The input electrical power, which was used to estimate the efficiency, was measured with the built-in meter of the motor. The experimental setup is presented in Fig. 2.

The parameters of the BEP were: $Q_{BEP} = 18.25$ m^3 h^{-1}, $H_{BEP} = 26.08$ m at the nominal (baseline) rotational speed of $n_n = 3355$ rpm determined with water.

Measurements

The reference performance curve was measured with water. Measurements were performed at least at eight operating points by each rotational speed. For glycerol solution, five different rota-

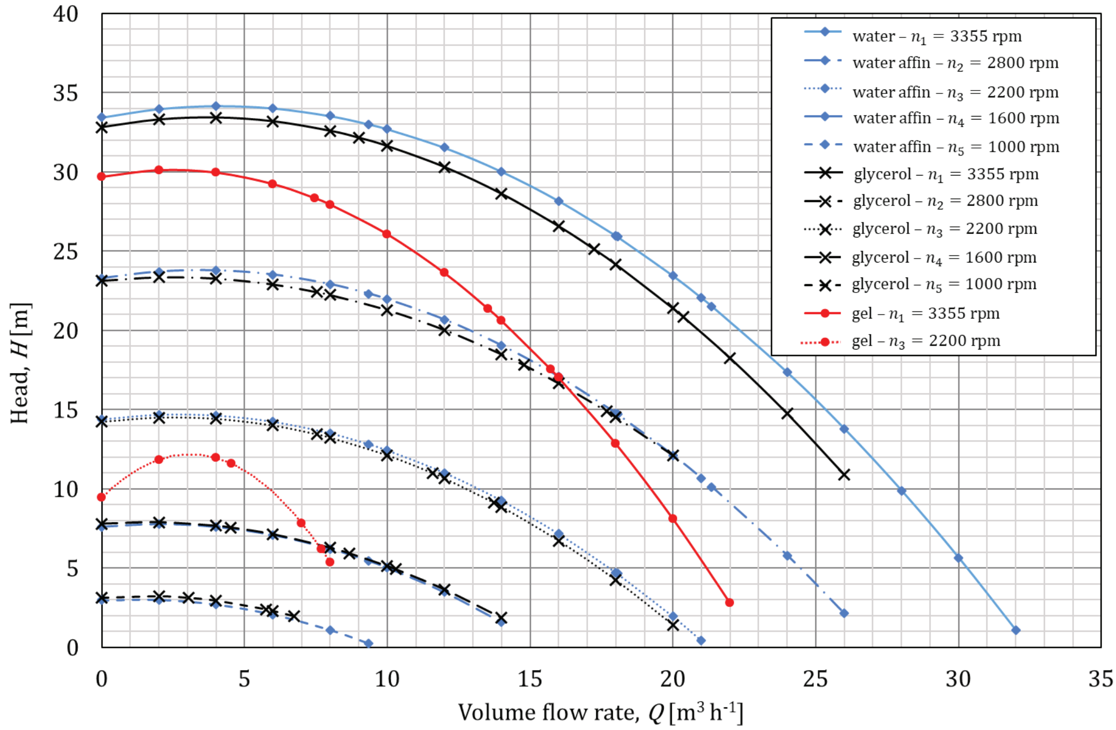

Fig. 3 – *Performance curves of the pump: measured at nominal speed and calculated with affinity-law for other rotational speeds with water (blue ◊); measured with glycerol (black ×) at five rotational speeds, and with gel (red ●) at two rotational speeds*

tional speeds were adjusted between the minimum (n_5 = 1000 rpm) and maximum (n_1 = 3355 rpm) values allowed by the pump control; while in the case of the gel –due to some unstable operation– only two speeds (n_3 = 2200 rpm and n_1 = 3355 rpm) were set. It took special care to avoid heating the fluids during the experiments, in order to keep the fluid at room temperature of 22 – 23 °C.

The measured baseline head was modelled with a quadratic polynomial equation, and the curves for other rotational speeds with water were calculated with the affinity laws[28]. The measured head curves as the function of the volume flow rate are shown in Fig. 3.

Prediction

ANSI/HI method

The ANSI/Hydraulic Institute method was used based on Pullum and Graham[23]. In this method, the parameter B is calculated from the BEP values at nominal rotational speed with the rheology as:

$$B = \frac{\left(\nu \cdot 10^6\right)^{0.5}\left(H_{\mathrm{BEP}}\right)^{0.0625}}{\left(Q_{\mathrm{BEP}} \cdot 3600\right)^{0.375} n_n^{0.25}} \tag{3}$$

where ν [m^2 s^{-1}] is the kinematic viscosity of the fluid; H [m] is the head; Q [m^3 s^{-1}] is the volume flow rate, and n_n [rpm] is the nominal rotational speed. The kinematic viscosity is estimated from the apparent dynamic viscosity μ_{app} [Pa s] and the density ρ [kg m^{-3}]: $\nu = \mu_{\mathrm{app}}/\rho$. This quantity is constant for the Newtonian glycerol, but depends on the shear rate for the non-Newtonian gel, as

$$\mu_{\mathrm{app}} = \frac{\tau}{\dot{\gamma}} = 4.796 \cdot \dot{\gamma}^{(0.3127-1)} \tag{4}$$

where τ [Pa] is the shear stress, and $\dot{\gamma}$ [s^{-1}] is the shear rate of the fluid.

The typical shear rate $\dot{\gamma}$ in Eq. (4) is obtained from the Rabinowitch-Mooney equation described for an equivalent pipe flow:

$$\dot{\gamma} = \left(\frac{3n'+1}{4n'}\right)\frac{8v_{\mathrm{eq}}}{D_{\mathrm{eq}}} \tag{5}$$

where the dimensionless n' is the local gradient of the curve $d\ln(\tau)/d\ln(8v_{\mathrm{eq}}/D_{\mathrm{eq}})$, v_{eq} [m s^{-1}] is the equivalent velocity, and D_{eq} [m] is the equivalent diameter. The latter is expressed with the impeller diameter D_{imp} [m] and parameter w [m], as $D_{\mathrm{eq}} = 4w\pi D_{\mathrm{imp}}/\{2(\pi D_{\mathrm{imp}}+w\}$. The equivalent velocity is determined from the volume flow rate with the following equation:

$$v_{\mathrm{eq}} = \frac{4Q}{\pi D_{\mathrm{eq}}^2} = \frac{4Q}{\pi\left(\frac{4w\pi D_{\mathrm{imp}}}{2\left(\pi D_{imp}+w\right)}\right)^2} \tag{6}$$

The correction factors for the volume flow rate ($C_{\mathrm{Q\_HI}}$) and the head ($C_{\mathrm{H\_HI}}$) in the ANSI/HI method are defined as:

$$C_{Q\_HI} = 2.71^{-0.165(\log B)^{3.15}} \tag{7}$$

$$C_{H\_HI} = 1 - \left[\left(1 - C_{Q\_HI}\right) \left(\frac{Q}{Q_{BEP}} \right)^{0.75} \right] \tag{8}$$

The predicted values of volume flow rate and head are derived from those with water as $Q'_{HI} = C_{Q\_HI} \cdot Q$ and $H'_{HI} = C_{H\_HI} \cdot H$.

In the ANSI/HI method, the parameter *w* is the questionable factor by application. This parameter of the actual pump with the investigated fluid has to be determined by regression, see the Results section.

Ofuchi method

In the Ofuchi method, the describing parameters of the pump operation have to be normalised. The relevant rotational specific speed (ω_s) for every operation point was:

$$\omega_s = \frac{2\pi n}{60} \frac{Q^{0.5}}{(gH)^{0.75}} \tag{9}$$

where n [rpm] is the actual rotational speed, Q [m^3 s^{-1}] is the actual volume flow rate, H [m] is the actual head, g [m s^{-2}] is the gravitational acceleration. With the values of the BEP for water at n_n nominal rotational speed, the ω_n normalised specific speeds can be calculated, along which the correction factors can be determined in the range of $0.6 \le \omega_n \le 1.25$ with the following:

$$\omega_n = \frac{\omega_s}{\omega_s\text{–BEP}} = \frac{n \dfrac{Q^{0.5}}{(H)^{0.75}}}{n_n \dfrac{Q_{BEP}^{0.5}}{(H_{BEP})^{0.75}}} \tag{10}$$

$$C_{H\_Of} = \frac{H'}{H} \left(\frac{n_n}{n} \right)^2 \Bigg|_{\omega_{n=const.}} \tag{11}$$

$$C_{Q\_Of} = \frac{Q'}{Q} \frac{n_n}{n} \Bigg|_{\omega_{n=const.}} \tag{12}$$

There is a simple correlation between the correction factors defined in Eq. (11) and Eq. (12) proposed by Stepanoff[28] and Ofuchi *et al.*[31] as:

$$C_{Q\_Of} = C_{H\_Of}^{1.5} \tag{13}$$

which assures that only the head correction factor has to be estimated from the baseline pump parameters. The head correction factor is the function of the modified Reynolds number:

$$C_{H\_Of} = Re_{mod}^{\left(\frac{-a}{Re_{mod}^{b}} \right)} \tag{14}$$

where a and b are constants. In this approach, the modified Reynolds number in Eq. (14) is defined as:

$$Re_{mod} = \frac{n}{n_n} \frac{Q_{BEP}}{Q^{0.5}} \frac{(gH)^{0.75}}{\nu (gH_{BEP})^{0.5}} \tag{15}$$

In practical application, the normalised specific speed with Eq. (9) and Eq. (10) can be calculated for each point of the baseline water curve. In the range of $0.6 \le \omega_n \le 1.25$, the modified Reynolds number can be determined with Eq. (15) for any rotational speed and fluid given by the kinematic viscosity value. Eq. (11) specifies the C_{H_Of} head correction factor, the one for the volume flow rate by Eq. (13). Finally, the points of the predicted degraded curves in the Ofuchi method were defined with Eq. (16) and Eq. (17):

$$Q'_{Of} = C_{Q\_Of} \cdot Q \cdot \left(\frac{n}{n_n} \right) \tag{16}$$

$$H'_{Of} = C_{H\_Of} \cdot H \cdot \left(\frac{n}{n_n} \right)^2 \tag{17}$$

Our study aimed to verify the polynomial relationship between the correction coefficients C_{Q_Of} and C_{H_Of} in Eq. (13) based on the experimental results. For this, the factors were calculated from the measured points at selected normalised specific speed values of $\omega_n = 0.6, 1, 1.25$. As shown in Fig. 4, the difference between the experimental values and the Eq. (13) was less than 0.5 %. Thus, the correspondence between the head and volume flow rate factors was verified with our pump. In addition, the good match with the literature also confirmed the accuracy of the measurements.

The new model of Ofuchi *et al.* is available for Newtonian fluids, but in non-Newtonian adaptation, the kinematic viscosity in Eq. (15) is not apparent. Moreover, the actual values of the constants in Eq. (14) are needed to be found.

Results

Estimated parameters

For the glycerol solution, the ANSI/HI method was directly applicable. In addition, the required parameter *w* was evaluated as a percentage of the impeller diameter in the ANSI/HI method for the power-law fluid. The best agreement between the measured and calculated curves for the actual pump with the power-law gel was obtained at 48 % of the impeller diameter, so the parameter was $w = 0.48 \cdot D_{imp}$. This value matched neither those proposed by Furlan *et al.*[25] nor the 25 % suggested by Pullum and Graham[23], but confirmed the findings

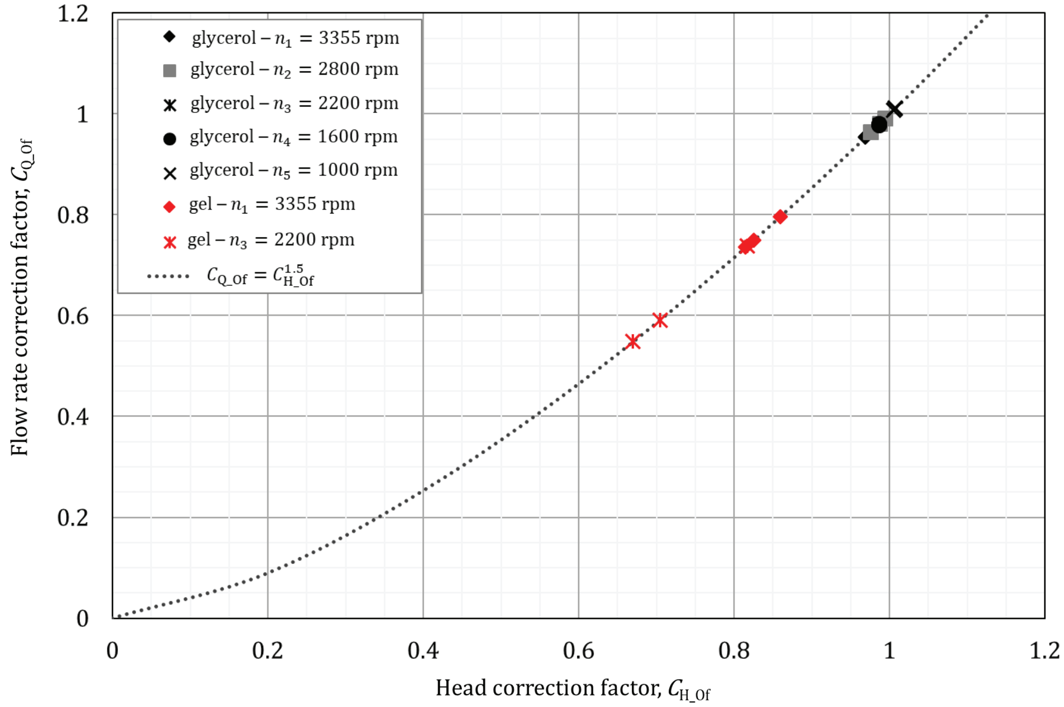

Fig. 4 – *Flow rate correction factors versus head correction factors by Ofuchi method at three normalised specific speed values with Wilo-Helix-EXCEL 1602-1/16/E/KS pump*

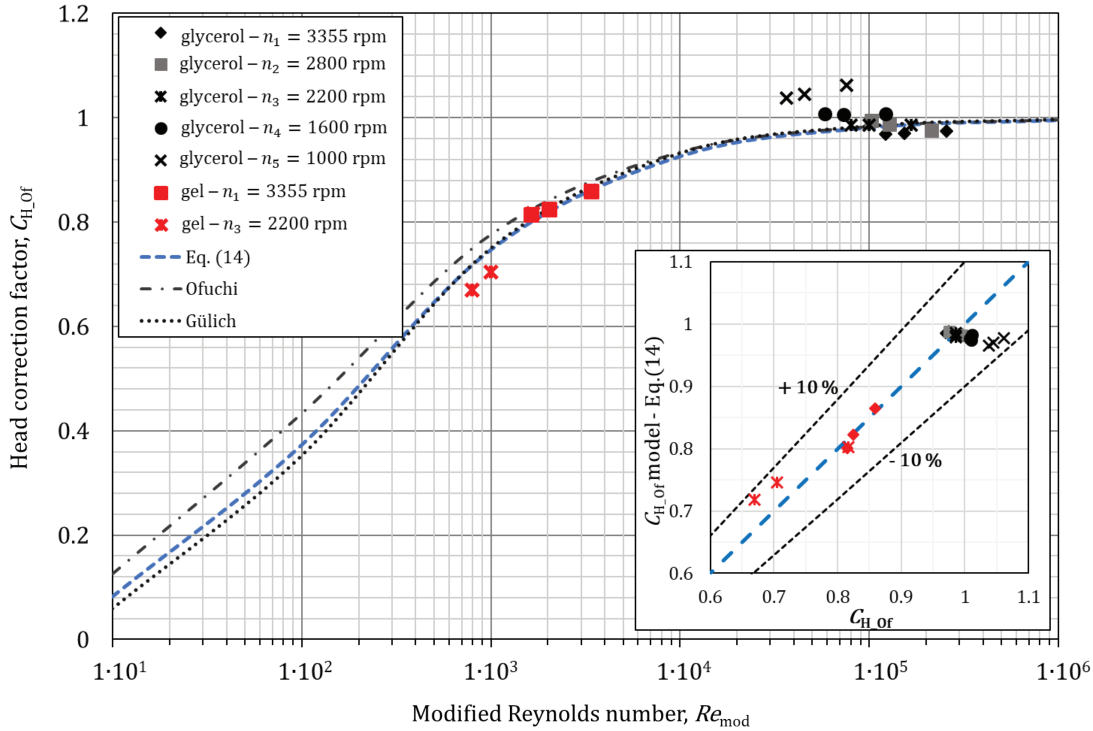

Fig. 5 – *Head correction factors versus modified Reynolds number by Ofuchi method at three normalised specific speed values with Wilo-Helix-EXCEL 1602-1/16/E/KS pump.*
Blue dashed curve: actual fit, black dash-dotted curve: Ofuchi's fit, dark grey dotted curve: Gülich's fit.

of Kalombo *et al.*[27] that it could be different for each pump with each fluid.

To adapt the Ofuchi method to non-Newtonian fluids, we assumed that an average shear rate could be used by estimating the gel's viscosity based on Metzner-Otto[22], as seen in Buratto *et al.*[26] We supposed a linear correlation between the average shear rate and the actual rotational speed of the pump[32]. The assumption $\dot{\gamma}\left[\mathrm{s}^{-1}\right] = c \cdot n\left[\mathrm{s}^{-1}\right]$ was tested at the baseline rotational speed n_n. The best fit was defined at the minimum of the mean square deviation between the measured and predicted curve, and resulted in constant c = 3.01 with a coefficient of determination of R^2 = 0.9971. This determined actual shear rate was used to estimate the apparent viscosity of the power-law fluid as in Eq. (4), and the derived kinematic viscosity was applied in Eq. (15).

The actual pump data with the different fluids at fixed normalised specific speed values were used to evaluate parameters a and b in Eq. (14) by the nonlinear least squares method, using MatLab. The values of the regression were found as a = 5.513 and b = 0.705 with the coefficient of determination of R^2 = 0.9691, which were very close to those suggested by Gülich[30] ($a_{\text{Gülich}}$ = 6.7 and $b_{\text{Gülich}}$ = 0.735), and Ofuchi *et al.*[31] (a_{Ofuchi} = 4.462 and b_{Ofuchi} = 0.695). The regression curve is presented in Fig. 5.

Predicted curves

The prediction with ANSI/HI method was performed in the entire range of the measured volume flow rates, but for the Ofuchi method, $\omega_n < 1.25$ is a mathematical upper limit of the application. It was also mentioned by Ofuchi *et al.*[31] that $0.1 \cdot Q_{BEP}$ instead of $0.6 < \omega_n$ as lower limit, yields good results in certain conditions, so we also used this value.

In the case of 25 % glycerol, the performance curves' shape changes were not remarkable, as shown in Fig. 6. The head curves predicted with both methods were in good agreement with the measured ones. Only the ANSI/HI model at rotational speed n_1 = 3355 rpm with high volume flow rate and both approaches at n_5 = 1000 rpm were outside the ±5 % error margin of the experimental data. The latter can be explained by the uncertainty of the measurements at low volume flow rates.

With the power-law gel, the measured head degradation was significant, and the shape of the experimental head curve differed from the baseline curve, as presented in Fig. 7. The ANSI/HI estimation did not result in performance curves similar to those measured. However, it was still within the ±5 % error band in that volume flow rate range, where the Ofuchi method is also valid.

The Ofuchi approach proved accurate at n_1 = 3355 rpm, and provided a good match with the experimental data with the average relative difference of 2 %. The Ofuchi curve was qualitatively better at the lower rotational speed than the ANSI/HI curve. Still, the mean relative difference was 10 %, and the highest value was 31 % between the experimental and calculated heads in the investigated range. It can also be noted that the particular degradation by the zero flow head cannot be modelled with this method due to the lower limit of application, as the method is valid only for $Q > 0.1 \cdot Q_{BEP}$[31].

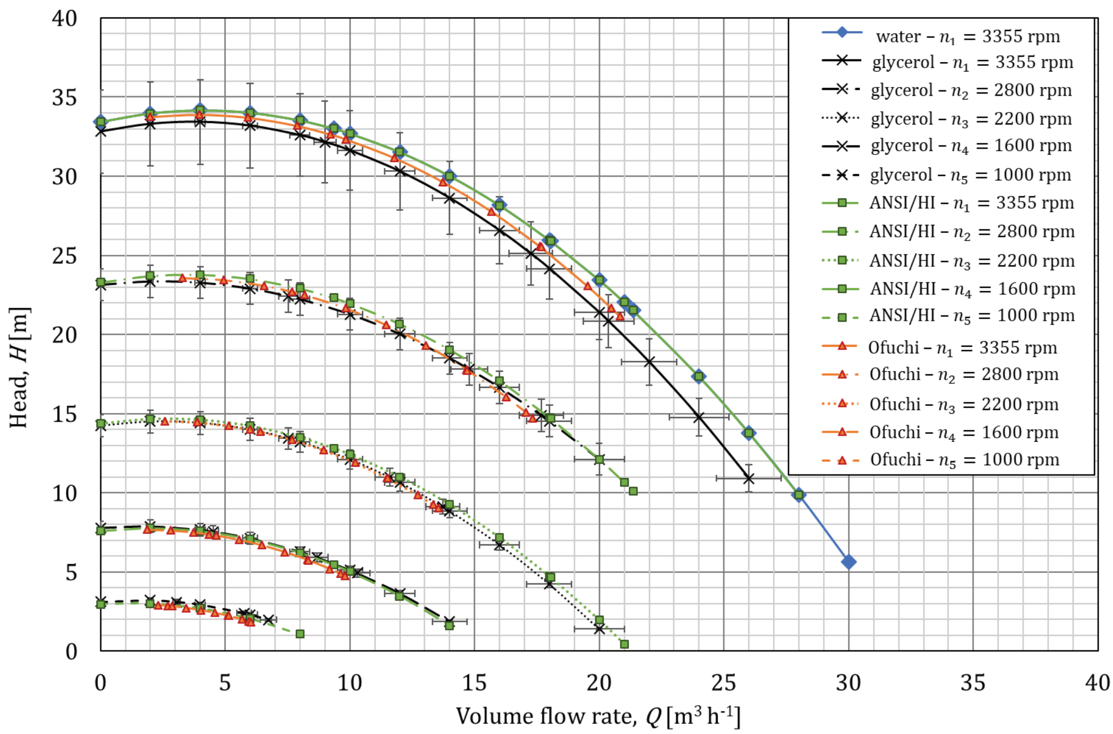

Fig. 6 – *Measured (black curves with ±5 % error) and predicted performance curves with glycerol at different rotational speeds with measured baseline curve with water as reference (thick blue with ◊). Green □: predicted with ANSI/HI method, orange Δ: predicted with Ofuchi method.*

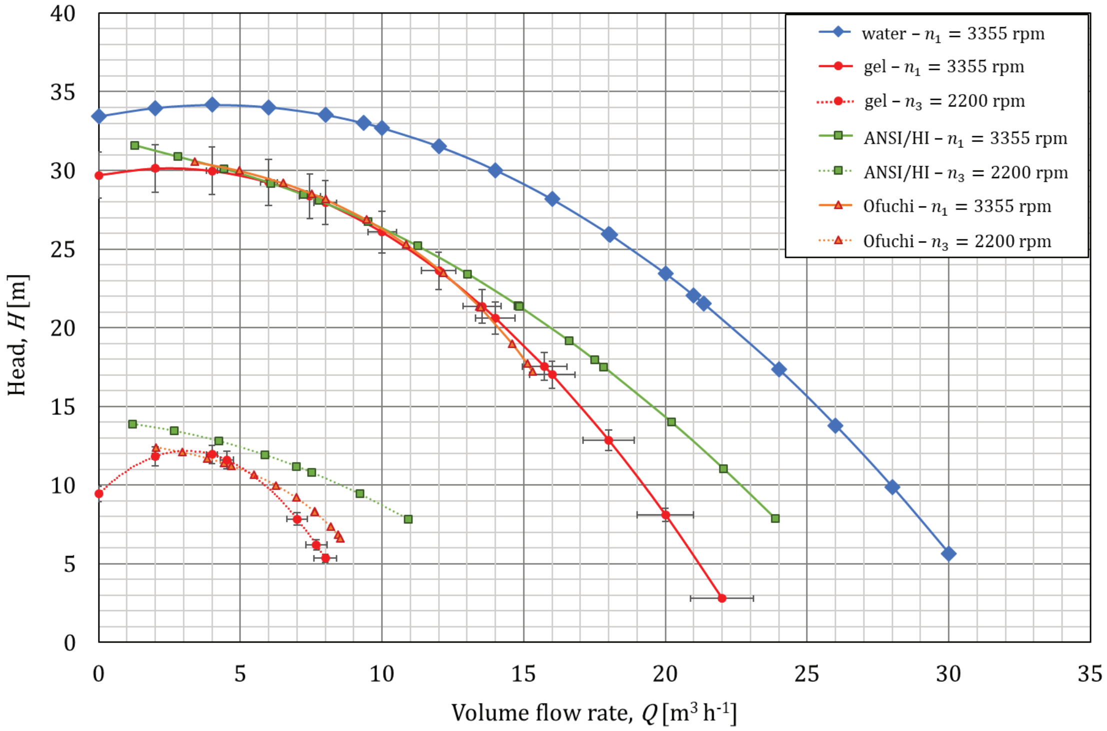

Fig. 7 – *Measured (red curves ● with ±5 % error) and predicted performance curves with gel at different rotational speeds with measured baseline curve with water as reference (thick blue with ◊). Green □: predicted with ANSI/HI method, orange Δ: predicted with Ofuchi method.*

Conclusions

The following conclusions can be drawn from the present study, which shows the results of comparing two prediction methods based on experiments carried out with a two-stage radial centrifugal pump with the specific speed of $n_q = 35$.

– With the viscous Newtonian test fluid, both the ANSI/HI and Ofuchi methods could estimate the head degradation of the radial centrifugal pump with sufficient accuracy of ±5 %.
– With the examined power-law fluid, the ANSI/HI model failed to replicate the measured curves, even though the key parameter *w* was estimated based on the experimental results.
– The characteristic parameters *a* and *b* of the Ofuchi method were successfully determined for our pump. To extend the prediction for non-Newtonian applications, the average shear rate was introduced to assess the apparent viscosity of the power-law gel.
– The predicted head degradation agreed quite well with the experiments at the nominal rotational speed with the mean difference of 2 %. The Ofuchi method predicted the performance curve qualitatively well, even at the lower rotational speed.
– Despite its limitations, this study has presented that, with properly chosen constants, the new model of Ofuchi *et al.* can be used to predict head degradation of pumps with pseudoplastic power-law fluids.

The work was limited regarding the pump type and the fluids, so several questions remain unanswered. Future research should be undertaken by experiments with other pumps and Bingham or Herschel-Bulkley fluids with yield stress.

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Nomenclature

a, b – constants in the Ofuchi method
B – parameter of the ANSI/HI method
BEP – best efficiency point
c – constant in the ANSI/HI method
C_{H_HI} – head correction factor in the ANSI/HI method
C_{H_Of} – head correction factor in the Ofuchi method
C_{Q_HI} – volume flow rate correction factor in the ANSI/HI method
C_{Q_Of} – volume flow rate correction factor in the Ofuchi method
D_2 – outlet impeller diameter, m
D_{imp} – impeller diameter, m
D_{eq} – equivalent diameter, m
g – gravity, m s^{-2}
H – head, m
H' – predicted head, m
n – actual rotational speed, rpm
n_n – nominal rotational speed, rpm
n_q – specific speed; $n_q = n_n Q_{BEP}^{0.5} \left(H_{BEP}/z\right)^{-0.75}$
n' – local gradient of the curve $\mathrm{d}\ln\left(\tau\right)/\mathrm{d}\ln\left(8v_{eq}/D_{eq}\right)$
Re_{mod} – modified Reynolds number
Q – volume flow rate, m^3 s^{-1}
Q' – predicted volume flow rate, m^3 s^{-1}
v_{eq} – equivalent fluid velocity, m s^{-1}
w – constant in the ANSI/HI method, m
z – number of stages

Greek symbols

$\dot{\gamma}$ – shear rate, s^{-1}
μ_{app} – apparent viscosity, Pa s
ν – kinematic viscosity, m^2 s^{-1}
ρ – density, kg m^{-3}
τ – shear stress, Pa
ω_n – normalised specific speed
ω_s – rotational specific speed
ϕ – flow number
ψ – pressure number

References

1. *Chhabra, R. P., Richardson, J. F.,* Non-Newtonian Flow and Applied Rheology: Engineering Applications, 2nd ed., Elsevier Butterworth-Heinemann, Oxford, UK, 2008, pp 10–11.
2. *Cabral, R. A. F., Telis, V. R. N., Park, K. J., Telis-Romero, J.,* Friction losses in valves and fittings for liquid food products, Food Bioprod. Process **89**(4) (2011) 375. doi: https://doi.org/10.1016/j.fbp.2010.08.002
3. *Telis-Romero, J., Telis, V. R. N., Yamashita, F.*, Friction factors and rheological properties of orange juice, J. Food Eng. **40**(1) (1999) 101. doi: https://doi.org/10.1016/S0260-8774(99)00045-X
4. *Telis-Romero, J., Thomaz, C. E. P., Bernardi, M., Telis, V. R. N., Gabas A. L.,* Rheological properties and fluid dynamics of egg yolk, J. Food Eng. **74**(2) (2006) 191. doi: https://doi.org/10.1016/j.jfoodeng.2005.01.044
5. *Atılgan, M. R., Unluturk, S.,* Rheological properties of liquid egg products (LEPS), Int. J. Food Prop. **11**(2) (2008) 296. doi: https://doi.org/10.1080/10942910701329658
6. *Ofuchi, E. M., Stel, H., Sirino, T., Vieira, T., Ponce, F., Chiva, S., Morales, R. E. M.,* Numerical investigation of the effect of viscosity in a multistage electric submersible pump, Eng. Appl. Comput. Fluid Mech. **11**(1) (2017) 258. doi: https://doi.org/10.1080/19942060.2017.1279079
7. *Jiang, S., Chen, X., Cao, G., Tan, Y., Xiao, X., Zhou, Y., Liu, S., Tong, Z., Wu, Y.,* Optimisation of fresh concrete pumping pressure loss with CFD-DEM approach, J. Con. Build. Mat. **276** (2021) 122204. doi: https://doi.org/10.1016/j.conbuildmat.2020.122204
8. *Csizmadia, P., Till, S.,* Investigation of activated sludge pumping through a case study, Circ. Econ. Environ. Prot. **4**(1) (2020) 5.
9. *Perse Slemenik, L., Zumer, M.,* Mixing and viscosity determinations with helical ribbon impeller, Chem. Biochem. Eng. Q. **18**(4) (2004) 363.
10. *Bombač, A., Žumer, M., Žun, I.,* Power consumption in mixing and aerating of shear thinning fluid in a stirred vessel, Chem. Biochem. Eng. Q. **21**(2) (2007) 131.
11. *Seyssiecq, I., Karrabi, M., Roche, N., In situ* rheological characterisation of wastewater sludge: Comparison of stirred bioreactor and pipe flow configurations, Chem. Eng. J. **259** (2015) 205. doi: https://doi.org/10.1016/j.cej.2014.07.102
12. *Polizelli, M. A., Menegalli, F. C., Telis, V. R. N., Telis-Romero, J.*, Friction losses in valves and fittings for power-law fluids, Brazilian J. Chem. Eng. **20**(4) (2003) 455. doi: https://doi.org/10.1590/S0104-66322003000400012
13. *Kfuri, S. L. D., Silva, J. Q., Soares, E. J., Thompson, R. L.,* Friction losses for power-law and viscoplastic materials in an entrance of a tube and an abrupt contraction, J. Pet. Sci. Eng. **76** (2011) 224. doi: https://doi.org/10.1016/j.petrol.2011.01.002
14. *Csizmadia, P., Hős, Cs.,* CFD-based estimation and experiments on the loss coefficient for Bingham and power-law fluids through diffusers and elbows, Comput. Fluids **99** (2014) 116. doi: https://doi.org/10.1016/j.compfluid.2014.04.004
15. *Bíbok, M., Csizmadia, P., Till, S.*, Experimental and numerical investigation of the loss coefficient of a 90° pipe bend for power-law fluid, Period. Polytech. Chem. Eng. **64**(4) (2020) 469. doi: https://doi.org/10.3311/ppch.14346
16. *Addie, G. R., Roudnev, A. S., Sellgren, A.,* The new ANSI/HI centrifugal slurry pump standard, J. South African Ins.t Min. Metall. **107**(6) (2007) 403.
17. KSB, Special Issues when Pumping Viscous Fluids in KSB, Selecting Centrifugal Pumps, 4th ed, KSB Aktiengeselschaft, Frankenthal, Germany, 2005, pp 48–53.
18. *Heywood, N., Slatter, P.,* Deration of Centrifugal Pumps for Non-Newtonian Slurries, in 19th International Conference on Transport and Sedimentation of Solid Particles, 2019, pp. 219–227.

19. *Aldi, N., Buratto, C., Casari, N., Dainese, D., Mazzanti, V., Mollica, F., Munari, E., Occari, M., Pinelli, M., Randi, S., Spina, P. R., Suman, A.,* Experimental and numerical analysis of a non-Newtonian fluids processing pump, Energy Procedia **126** (2017) 762.
doi: https://doi.org/10.1016/j.egypro.2017.08.247
20. *Walker, C. I., Goulas, A.,* Performance characteristics of centrifugal pumps when handling non-Newtonian homogeneous slurries, Proc. Instn. Mech. Engrs. **198A**(1) (1984) 41.
doi: https://doi.org/10.1243/PIME_PROC_1984_198_006_02
21. *Sery, G., Kabamba, B., Slatter, P.,* Paste pumping with centrifugal pumps: Evaluation of the Hydraulic Institute Chart De-Rating Procedures, Proc Ninth Int Semin Paste Thick Tailings, 2006, pp 403–412.
doi: https://doi.org/10.36487/acg_repo/663_35
22. *Metzner, A. B., Otto, R. E.,* Agitation of non-Newtonian fluids, AIChE J. **3**(1) (1957) 3.
doi: https://doi.org/https://doi.org/10.1002/aic.690030103
23. *Graham, L. J. W., Pullum, L., Slatter, P., Sery, G., Rudman, M.,* Centrifugal pump performance calculation for homogeneous suspensions, Can. J. Chem. Eng. **87**(4) (2009) 526.
doi: https://doi.org/10.1002/cjce.20192
24. *Irgens, F.,* Basic Equations in Fluid Mechanics, in Rheology and Non-Newtonian Fluids, Springer International Publishing Switzerland, 2013, pp 25–61.
doi: https://doi.org/10.1007/978-3-319-01053-3
25. *Furlan, J., Visintainer, R., Sellgren, A.,* Centrifugal pump performance when handling highly non-Newtonian clays and tailings slurries, Can. J. Chem. Eng. **94**(6) (2016) 1108.
doi: https://doi.org/10.1002/cjce.22495
26. *Buratto, C., Occari, M., Aldi, N., Casari, N., Pinelli, M., Spina, P. R., Suman, A.,* Centrifugal pumps performance estimation with non-Newtonian fluids: Review and critical analysis, Proceedings of 12th Eur Conf Turbomach Fluid Dyn Thermodyn ETC, 2017.
doi: https://doi.org/10.29008/etc2017-248
27. *Kalombo, J. J. N., Haldenwang, R., Chhabra, R. P., Fester, V. G.,* Centrifugal pump derating for non-Newtonian slurries, J. Fluids Eng. Trans. ASME **136**(3) (2014) 1.
doi: https://doi.org/10.1115/1.4025989
28. *Stepanoff, A. J.,* Pumping Viscous Liquids with Centrifugal Pumps, in Centrifugal and Axial Flow Pumps: Theory, Design, and Application, 2nd ed., Wiley, New York, 1957, pp 308–324.
29. *Ofuchi, E. M., Stel, H., Vieira, T. S., Ponce, F. J., Chiva, S., Morales, R. E. M.,* Study of the effect of viscosity on the head and flow rate degradation in different multistage electric submersible pumps using dimensional analysis, J. Pet. Sci. Eng. **156** (2017) 442.
doi: https://doi.org/10.1016/j.petrol.2017.06.024
30. *Gülich, J. F.,* Pumping highly viscous fluids in Centrifugal Pumps, 2nd ed, Springer Berlin, Heidelberg, 2010, pp 741–757.
doi: https://doi.org/10.1007/978-3-642-12824-0
31. *Ofuchi, E. M., Cubas, J. M. C., Stel, H., Dunaiski, R., Vieira, T. S., Morales, R. E. M.,* A new model to predict the head degradation of centrifugal pumps handling highly viscous flows, J. Pet. Sci. Eng. **187** (2020) 106737.
doi: https://doi.org/10.1016/j.petrol.2019.106737
32. *Csizmadia, P., Lukácsi, D. L., Till, S.*, Prediction of Head Degradation of a Centrifugal Pump Handling Power-Law Fluid, In: Proceedings of the 2nd International Conference on Fluid Flow and Thermal Science (ICFFTS'21), **105** (2021) 1.
doi: https://doi.org/10.11159/icffts21.105